Viewpoint

Negative Frequencies Get Real

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A new resonant emission component from solitons that had been ignored has now been identified and studied.

Subject Areas: **Optics**

A Viewpoint on:
Negative-Frequency Resonant Radiation
E. Rubino, J. McLenaghan, S. C. Kehr, F. Belgiorno, D. Townsend, S. Rohr, C. E. Kuklewicz, U. Leonhardt, F. König, and D. Faccio
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A soliton is a localized "lump" of light that is the product of wave effects in a nonlinear medium and can, under certain conditions, emit low-intensity, positive frequency resonant radiation in its wake, due to the phase matching between its momentum and the dispersion of the medium itself. Writing in *Physical Review Letters*, Eleonora Rubino at the University of Insubria in Como, Italy, and collaborators have discovered that there should be a negative frequency counterpart of this resonant emission, which they have identified experimentally in two different systems [1].

When light travels through a medium, the dispersion—the relation between frequency and momentum of a wave—has to be taken into account. This has very important consequences: vacuum, for example, possesses a trivial dispersion—a straight line across all frequencies—and thus all colors travel at the same speed in empty space. However, in any other medium, for example, a silica optical fiber, the dispersion is far from being a straight line, so that different frequencies travel at different velocities. This produces a typical temporal broadening of short input pulses in fibers. When nonlinear effects also come into play, the momentum (and thus the refractive index) depends not only on frequency, but also on the intensity of light. In this case, the spreading due to dispersion, and the self-focusing effect due to nonlinearity, can perfectly balance to create solitons—localized bell-shaped waves that travel for very long distances in the waveguide without any distortion. Solitons are nowadays commonly produced, sometimes in large quantities, in many experiments [2].

The soliton momentum is nonlinear and depends on its intensity. When it and fiber dispersion coincide, the so-called phase matching takes place. Phase matching is a very common phenomenon in nonlinear optics, when two or more waves at different frequencies are allowed to exchange energy efficiently, due to the coincidence of their phases (that are proportional to their momenta). Under phase-matching conditions, a special kind of low-intensity radiation can be emitted by the soliton at a well-defined frequency, called resonant radiation [3]. This radiation is one of the essential ingredients of the supercontinuum generation, an extremely important and useful nonlinear phenomenon, which massively broadens the spectrum of an input narrow-band pulse, producing a flat spectral distribution over a broad range of frequencies, similar to sunlight, but coherent, and more intense by six orders of magnitude [4].

Supercontinuum generation has been intensively studied in optical fibers over the last fifteen years, and thus the theoretical, analytical, and numerical tools that are available today are very adequate and advanced, and simulations that perfectly reproduce experimental findings are commonplace in any serious nonlinear optics laboratory [5]. It is therefore with great surprise that a missing ingredient of the supercontinuum generation has been recently identified experimentally, and explained theoretically, by Rubino *et al.*[1], in which the phase matching occurs between a soliton momentum and the fiber dispersion at negative frequencies.

It is the usual practice when dealing with the classical Maxwell equations, to assume that only positive frequencies have an acceptable physical meaning. When the soliton dispersion (which is basically a straight line with a slope proportional to its velocity) and the fiber dispersion (which is a rather complicated curve) are phase matched at positive frequencies, *positive* resonant radiation is produced, which is the one that most people observe in experiments. However, there is no particular reason why we have to restrict our attention to positive frequencies only, since any electromagnetic wave is a real field, and thus it is the sum of a field with positive frequencies and its

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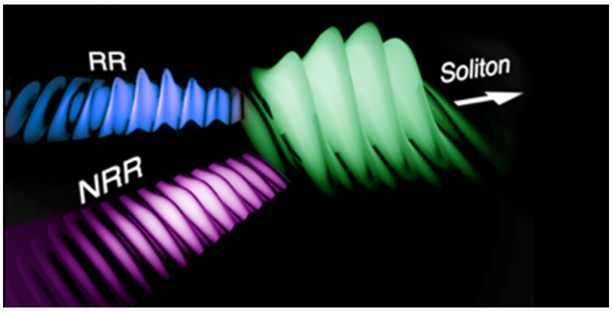

FIG. 1: Schematic representation of a propagating optical soliton that sheds in its wake two distinct blue-shifted modes: the usual positive resonant radiation (RR), and a second mode identified by Rubino et al. as negative resonant radiation (NRR). All these modes are traveling in the forward direction indicated by the arrow. (APS/Alan Stonebraker)

complex conjugate field, and therefore possesses negative frequencies.

This simple reasoning leads to a phase matching between the soliton and the negative frequency part of the fiber dispersion, and the curious, but logical, consequence is that this phase matching is asymmetric, and so leads to the generation of a new resonant radiation peak at a frequency that is not mirror symmetric with its positive energy counterpart.

Any physical electric field is a real function, and therefore can be expressed as a sum of two complex functions (called envelopes), which are conjugates of each other. If the first complex function contains only positive frequencies, the second must contain only negative ones. These two pieces always come together, and thus negative frequencies have always been thought to be “redundant,” i.e., positive and negative frequencies should contain the same physics in classical electromagnetism. However, the point of Rubino’s work is that this is not true. The presence of a soliton (or, as a matter of fact, any wave that has a steep front of intensity) can break the symmetry of the phase-matching condition, thus leading to two different resonant radiation frequencies, one which is positive (shown as RR in Fig. 1) and the other which is negative (shown as NRR in Fig. 1). The analysis shows that these two frequencies have different magnitudes, as well as different signs. Nevertheless, since in the electric field every wave comes together with its complex conjugate, at the end, the negative frequency mode instantaneously acquires a positive frequency by switching its sign, and thus in the experiments, one should see not only the conventional RR, but the NRR as well, although the latter must have a smaller amplitude than the former. In 2008, a related and very similar phenomenon was demonstrated in water waves propagating near an “event horizon” [6].

In order to prove the existence of this negative-frequency resonant radiation, which is typically emitted at shorter wavelengths than its positive counterpart, the team has performed experiments with photonic crystal fibers (PCFs)—highly nonlinear fibers in which the formation of solitons and resonant radiation is particularly favorable [7]. They launched extremely short pulses of 7 femtoseconds (fs) into a 5-mm PCF with a very broad input spectrum that favors the energy transfer between the soliton and the negative resonant radiation, which they were able to observe directly, exactly at the predicted frequency. They repeated a similar experiment in a bulk medium (2 cm of calcium fluoride), using 60 fs input Bessel pulses, again demonstrating the formation of a small-amplitude negative resonant radiation at the predicted wavelength [5].

The above findings, when and if confirmed experimentally by other groups, could generate a renewed interest in supercontinuum generation, introducing a novel and refreshing point of view on this “old” phenomenon. If researchers manage to control the formation and the generation of the negative resonant radiation, there will be chances to push supercontinuum generation to shorter and shorter wavelengths, which will be very useful for several applications, such as optical coherence tomography, the characterization of optical devices, and the generation and measurement of frequency combs. The work could affect substantially phenomena in other fields that are described by the nonlinear Schrödinger equation, for example, the formation of Bose-Einstein condensates. Rubino *et al.* claim that the generation of these new radiation bands cannot be explained in any other known way by only taking into account the “conventional” positive frequencies. In future experiments based on optical fibers, to conclusively prove the relevance of this “negative world” in nonlinear optics, it will be especially important to exclude the positive resonant radiation frequencies that are due to phase matching between the soliton and higher-order linear modes in fibers [8], the so-called four-wave mixing between solitons and continuous waves [9], and the generation of purely positive frequencies from dispersive moving fronts [10], while in bulk crystals one must exclude the contribution of higher-order Bessel-Gauss states [11], which are all potentially able to produce low-intensity waves at wavelengths close to those predicted in this study.

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About the Author

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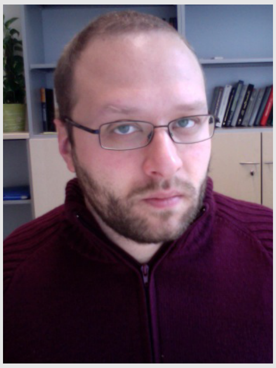

Fabio Biancalana has a master's degree in Particle Physics from the University of Rome III, Italy, and received his Ph.D. in nonlinear fiber optics from the University of Bath, UK. He enjoyed several fellowships in Ireland and Wales before becoming the leader of the independent theory group "Nonlinear Photonic Nanostructures" at the Max Planck Institute for the Science of Light in Erlangen, Germany. His main interests in research are photonics crystal fibers, optical solitons, and nonlinear optics in general.

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