Treating Epidemics as Feedback Loops

A new model of epidemics describes infections as part of a feedback loop—an approach that might one day help optimize interventions such as social distancing and lockdowns.

By Andrea Parlangeli

During the worst days of the COVID-19 pandemic, many of us became accustomed to news reports on the reproduction number *R*, which is the average number of cases arising from a single infected case. If we were told that *R* was much greater than 1, that meant the number of infections was growing rapidly, and interventions (such as social distancing and lockdowns) were necessary. But if *R* was near to 1, then the disease was deemed to be under control and some relaxation of restrictions could be warranted. New mathematical modeling by Kris Parag from Imperial College London shows limitations to using *R* or a related growth rate parameter for assessing the "controllability" of an epidemic [**1**]. As an alternative strategy, Parag suggests a framework based on treating an epidemic as a positive feedback loop. The model produces two new controllability parameters that describe how far a disease outbreak is from a stable condition, which is one with feedback that doesn't lead to growth.

During an epidemic, interventions such as face masks and quarantines can help limit the spread of a disease. A new model helps to assess how well these measures are working at controlling the epidemic.
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Parag's starting point is the classical mathematical description of how an epidemic evolves in time in terms of the reproduction number *R*. This approach is called the renewal model and has been widely used for infectious diseases such as COVID-19, SARS, influenza, Ebola, and measles. In this model, new infections are determined by past infections through a mathematical function called the generation-time distribution, which describes how long it takes for someone to infect someone else. Parag departs from this traditional approach by using a kind of Fourier transform, called a Laplace transform, to convert the generation-time distribution into periodic functions that define the number of the infections. The Laplace transform is commonly adopted in control theory, a field of engineering that deals with the control of machines and other dynamical systems by treating them as feedback loops.

The first outcome of applying the Laplace transform to epidemic systems is that it defines a so-called transfer function that maps input cases (such as infected travelers) onto output infections by means of a closed feedback loop. Control measures (such as quarantines and mask requirements) aim to disrupt this loop by acting as a kind of "friction" force. The framework yields two new parameters that naturally describe the controllability of the system: the gain margin and the delay margin. The gain margin quantifies how much infections must be scaled by interventions to stabilize the epidemic (where stability is defined by $R = 1$). The delay margin is related to how long one can wait to implement an intervention. If, for example, the gain margin is 2 and the delay margin is 7 days, then the epidemic is stable provided that the number of infections doesn't double and that control measures are applied within a week. In general, outbreaks with smaller margins necessitate more control effort.

Parag shows that his method has the advantage of providing reliable predictions in cases where the traditional indicator R fails. Indeed, in real epidemics, many cases often go undetected, as some infected individuals never exhibit observable symptoms and are therefore not subjected to targeted measures such as quarantines. "The controllability of an epidemic is strongly influenced by the untargeted group, which is not controlled but still able to spread the disease," Parag says. The effect of this invisible group has been considered before, but Parag's approach better defines the threshold in the size of the group beyond which targeted controls will fail. "Control measures only work if the untargeted portion is not too detrimental to the whole system," he says. If the situation gets out of control, more drastic measures such as a lockdown must be taken.

Like every mathematical model, Parag's model is based on assumptions and is therefore limited in the types of situations it can be applied to. First of all, it is based on linear equations, which means that it is only valid during the early period of an epidemic, when growth is exponential and there are not yet saturation effects coming from part of the population being immunized from previous infections. Secondly, the model works only for interventions that are implemented continuously in time (such as quarantines) but not for those that turn on suddenly (such as lockdowns). Finally, the model is deterministic and thus does not include random effects.

Despite these limitations, Alfio Quarteroni, an applied mathematician from the Swiss Federal Institute of Technology in Lausanne (EPFL) and the Polytechnic University of Milan, thinks that this work is a core contribution to the still-developing field of epidemic controllability. "It is a unified framework for epidemics based on a positive feedback loop approach, which can be crucial during outbreaks to evaluate different control measures," he says. The approach offers two new metrics, the gain and the delay margins, that, in principle, can outperform the standard controllability approach, Quarteroni says. "A validation of the results in a real case epidemic scenario would be very welcome."

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REFERENCES

1. K. V. Parag, "How to measure the controllability of an infectious disease?" **Phys. Rev. X 14, 031041 (2024).**