

Quantum Probability Requires Probability

A new analysis reveals that all major derivations of the Born rule depend on a probabilistic assumption.

By **John B. DeBrot**

Could the probabilities of quantum mechanics have a deterministic origin? The desire to give a nonprobabilistic explanation of quantum probabilities is one reaction to the infamous measurement problem: Why does a quantum system evolve continuously and deterministically when it's not observed but discontinuously and randomly once it is? Probability appears in only one postulate of quantum theory, the Born rule. If the Born rule could be derived without introducing a probabilistic assumption in the first place, it would furnish an appealing path to solving the measurement problem—the stochastic nature of measurement would have a nonstochastic explanation. However, a new analysis by Jiaxuan Zhang [1], a graduate student at the University of Oxford in the UK, shows that a particular probabilistic assumption is indispensable in every major derivation of the Born rule, casting fundamental doubt on this hope.



Figure 1: Among the key ingredients necessary to derive the Born rule, Zhang observes that additivity is indispensable in existing derivations.

Credit: Carin Cain/APS

The Born rule translates the quantum state of a physical system and the operators representing a measurement into probabilities for the possible outcomes. As it is the only part of the standard formalism where probabilities appear, it is at the center of the debate about whether nature provides an intrinsic source of indeterminism. A large fraction of physicists favor the Copenhagen interpretation of quantum mechanics, associated with Niels Bohr, Werner Heisenberg, and other founders of quantum theory [2]. Many subscribers to Copenhagen and related views expect indeterminism to be irreducible on the grounds that quantum measurement differs fundamentally from classical conceptions. Members of this class do not expect indeterminism to vanish in any further unpacking of the postulates. By contrast, another broad class of interpretations, perhaps best characterized by the many-worlds interpretation (MWI), considers indeterminism an imperfection, to be buffed out by a more complete understanding. To members of this class, an invocation of probability calls for further explanation.

Although some maintain the hope that the nonprobabilistic postulates of quantum theory alone will suffice to derive the Born rule, all existing derivations involve additional assumptions [3]. Nevertheless, supporters of MWI and related interpretations would be pleased if a set of postulates could be entirely nonprobabilistic. Decades of derivations exist, spanning distinct streams of physical intuition. Some are overtly probabilistic, whereas others claim to weaken or even eliminate probabilistic dependence. Their utilization of many subtly different postulates makes comparisons difficult. What is most needed is not another derivation but rather an analysis of what existing derivations share. This is what Zhang provides.

Her approach unfolds from the observation that several

common assumptions employed in Born-rule derivations are often not clearly distinguished by their authors. This failure can obscure the origin of probability in Born-rule derivations. By scrutinizing underlying assumptions more carefully, Zhang identified sources of confusion in existing approaches and isolated what is truly necessary in a given derivation.

Any derivation of the Born rule involves applying a set of assumptions, or postulates, to a general domain of possible rules until all rules but one are excluded. As Zhang's purpose is to diagnose which postulates are necessary to obtain quantum probabilities, she does not presuppose any prior probabilistic structure. Instead, to represent a measurement, she uses a function that maps a quantum state and a measurement operator (the mathematical object representing a specific outcome) to a real number. She then demonstrates that three assumptions about the functions that commonly appear in Born-rule derivations—additivity, noncontextuality, and normalization—are logically independent (Fig. 1).

Additivity requires that the total value of mutually exclusive alternatives equals the sum of their individual values. Noncontextuality means that an outcome's value doesn't change depending on the measurement context; for example, on the nature of the measurement or on what other measurements are made alongside it. Normalization is a scaling rule setting the total value of all possible outcomes to one.

Of these, additivity over any countable set of compatible measurement operators does the bulk of the probabilistic heavy lifting and, unlike normalization or noncontextuality, is considered an intrinsically probabilistic postulate. What's more, standard arguments show that additivity is required for a self-consistent quantification of uncertainty over mutually exclusive alternatives [4, 5]. Probabilities are additive by necessity, while normalized only by convention, and, as Zhang clarifies, they are independent of any noncontextuality assumptions. Thus, any hoped-for derivation of the Born rule from nonprobabilistic postulates must derive additivity rather than assume it. Yet Zhang shows that countable additivity is indispensable in all major derivations. No weaker form will suffice.

Attempts to sidestep additivity seem to either unwittingly invoke it through assumptions that turn out to be stronger or

rely on it implicitly through forms of additivity embedded in other assumptions. For example, an assumption used by Fabrizio Logiurato and Augusto Smerzi [6], which Zhang calls strong normalization, is actually equivalent to assuming both normalization and additivity. Conversely, she shows that additivity alone does not imply any sort of noncontextuality and that confusion on this point arises because a stronger form of noncontextuality is often implicitly assumed along with additivity.

With sharpened definitions in hand, Zhang analyzes five major Born-rule derivations spanning three derivation strategies. In Gleason-type derivations [7, 8], countable additivity is transparently assumed. In Deutsch-type derivations, including David Deutsch's original [9] and a derivation by Wojciech Zurek that strives to deliver the Born rule purely from the physical principles of symmetry and entanglement [10], countable additivity is shown to be necessary. Deutsch's decision-theoretic derivation implicitly assumes it, whereas Zurek's attempts use an explicitly weakened additivity assumption. This weakening, however, fails to keep probabilities mathematically smooth, allowing erratic jumps that undermine the Born rule [11]. Finally, Finkelstein-Hartle-type derivations are shown to be inconsistent unless countable additivity is added as a postulate [12, 13].

Prospects appear to be slim that we will see a derivation of the Born rule that does not, explicitly or implicitly, rely on an intrinsically probabilistic postulate. Indeed, Zhang regards her results as supporting the view that additional probabilistic postulates should be introduced to derive the Born rule. Of course, no impossibility proof has been given, and the identification of additivity as the seemingly necessary postulate may inform new attempts to do without it. Nevertheless, adherents of interpretations that center measurement and probability may read these results as a vindication: Nobody has gotten probabilistic content out without somewhere putting it in.

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REFERENCES

1. J. Zhang, "Summing to uncertainty: On the necessity of additivity in deriving the Born rule," *Phys. Rev. A* **114**, 042208

- (2026).
2. E. Gibney, “Physicists disagree wildly on what quantum mechanics says about reality, *Nature* survey shows,” *Nature* **643**, 1175 (2025).
 3. L. Vaidman, “Derivations of the Born rule,” in *Quantum, Probability, Logic* (Jerusalem Studies in Philosophy and History of Science), edited by M. Hemmo and O. Shenker (Springer, Cham, 2020), p. 567.
 4. R. T. Cox, *The Algebra of Probable Inference*. (Johns Hopkins Press, Baltimore, 1961).
 5. B. de Finetti, *Theory of Probability: A Critical Introductory Treatment.*, Vol. 1 (Wiley, New York, 1974).
 6. F. Logiurato and A. Smerzi, “Born rule and noncontextual probability,” *J. Mod. Phys.* **03**, 1802 (2012).
 7. A. Gleason, “Measures on the closed subspaces of a Hilbert space,” *J. Math. Mech.* **6**, 885 (1957).
 8. P. Busch, “Quantum states and generalized observables: A simple proof of Gleason’s theorem,” *Phys. Rev. Lett.* **91**, 120403 (2003).
 9. D. Deutsch, “Quantum theory of probability and decisions,” *Proc. R. Soc. London, Ser. A* **455**, 3129 (1999).
 10. W. H. Zurek, “Probabilities from entanglement, Born’s rule $p_k = |\psi_k|^2$ from envariance,” *Phys. Rev. A* **71**, 052105 (2005).
 11. H. Barnum, “No-signalling-based version of Zurek’s derivation of quantum probabilities: A note on “environment-assisted invariance, entanglement, and probabilities in quantum physics,” *arXiv: quant-ph/0312150* [quant-ph].
 12. D. Finkelstein, “Section of physical sciences: The logic of quantum physics,” *Trans. N. Y. Acad. Sci.* **25**, 621 (1963).
 13. J. B. Hartle, “Quantum mechanics of individual systems,” *Am. J. Phys.* **36**, 704 (1968).