Viewpoint

Fractional quantum Hall effect without Landau levels

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Researchers develop models that could exhibit a fractional quantum Hall effect in the absence of an external magnetic field.

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A Viewpoint on:
High-Temperature Fractional Quantum Hall States
Evelyn Tang, Jia-Wei Mei, and Xiao-Gang Wen
Physical Review Letters ***106****, 236802 2011* – Published June 6, 2011

Fractional Quantum Hall States at Zero Magnetic Field
Titus Neupert, Luiz Santos, Claudio Chamon, and Christopher Mudry
Physical Review Letters ***106****, 236804 2011* – Published June 6, 2011

Nearly Flatbands with Nontrivial Topology
Kai Sun, Zhengcheng Gu, Hosho Katsura, and S. Das Sarma
Physical Review Letters ***106****, 236803 2011* – Published June 6, 2011

The quantum Hall effect (QHE) is the remarkable observation of quantized transport in two dimensional electron gases placed in a transverse magnetic field: the longitudinal resistance vanishes while the Hall resistance is quantized to a rational multiple of h/e^2. The theory of the QHE is built largely around the special properties of single-particle free-electron states in a magnetic field—the celebrated Landau levels. This is particularly true of the F(ractional)QHE, where the construction of model wave functions with built-in analyticity forced by a restriction to the lowest Landau level has played an extremely important role in the theoretical development. Now, in three papers published back-to-back in *Physical Review Letters*[1–3], three research groups develop lattice models lacking this Landau level structure whose realizations could, in principle, exhibit a FQHE in the absence of an external magnetic field.

While the simplicity of the setting in which the QHE was first discovered was extremely helpful in unraveling its explanation, condensed matter physicists have periodically returned to the challenge of generalizing the reach of the phenomenon itself. There are two specific questions that have focused their attention. First, in the standard electron gases exhibiting the QHE, the surrounding solid has a fairly modest effect on their properties, which can be captured by a change in the (effective) mass of the electrons away from its value in free space. Does the QHE survive when the solid affects electronic motion more seriously and one needs to take the formation of energy bands into account? Second, a uniform magnetic field does two things: it breaks time reversal symmetry, but it also affects electron dynamics at long wavelengths in a specific fashion, as captured in the formation of Landau levels. Are both essential for the QHE?

The answer to both questions is known for the I(nteger)QHE, which is mostly a single-particle phenomenon. In a landmark paper in 1982, Thouless, Kohmoto, Nightingale, and den Nijs [4] analyzed the uniform-field Hall effect in a strong periodic potential that was known to lead to an intricate spectrum, the so-called Hofstadter butterfly (see Fig. 1); they showed that it gives rise to an integer QHE under certain conditions, i.e., whenever the chemical potential lies in a gap. Indeed, the Hall conductance was shown to map to a topological invariant associated with filled bands—the (first) Chern number. Six years later, in another striking development, Haldane [5] answered the second question, showing by an explicit construction of a tight-binding model on a honeycomb lattice that a quantized Hall conductance can arise from a fully filled band even in the absence of a net magnetic field. In his model, time-reversal symmetry is broken by a spatially inhomogeneous mag-

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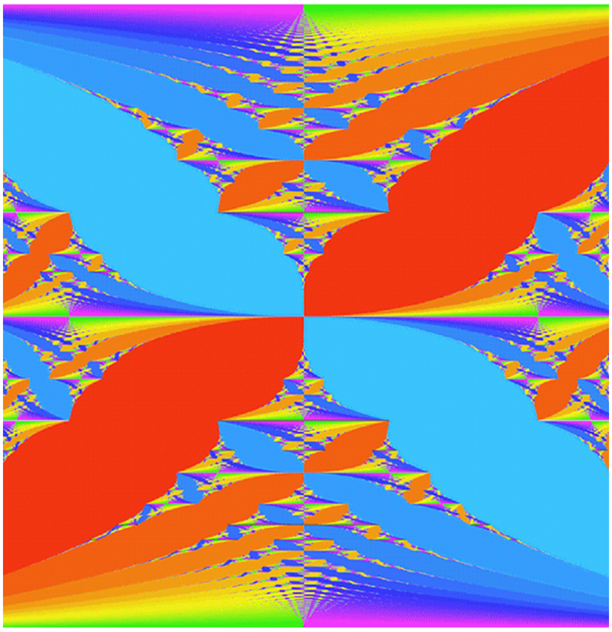

FIG. 1: A colored Hofstadter butterfly: This figure represents the phase diagram of Bloch electrons in a uniform magnetic field. The horizontal axis indicates the chemical potential and the vertical axis the flux through the system. Each color corresponds to a distinct topological phase with a particular quantized value of the Hall conductance. (Credit: Yosi Avron)

netic field with zero average, and the Hall conductance again equals the Chern number of the band.

The three recent papers [1–3] take up this development and address the next logical question: Can the FQHE, canonically a property of interacting electrons in a fractionally filled Landau level, also be separated from the weak lattice and uniform magnetic field limit? More precisely, they ask the following: If it is true for independent electrons that a filled Chern band is equivalent to a filled Landau level, then is it also true for interacting electrons that a fractionally filled Chern band is equivalent to a fractionally filled Landau level?

A Landau level involves a set of exactly degenerate single-particle states and thus, at a fractional filling, the kinetic energy alone does not select a ground state, but instead, it falls to the interactions to force the issue. By contrast, a Chern band typically will have a significant dispersion that will select a unique kinetic-energy-dominated ground state at reasonable interaction strengths, as it does in all metals. Recognizing this, all three papers devote considerable effort to constructing lattice models with nearly flat (degenerate) Chern bands. Neupert *et al.*[2] construct a flattened version of Haldane's model on a square lattice. They note that while a fully flattened model requires the inclusion of electron hopping over arbitrarily large distances, the hopping amplitudes decrease exponentially, which allows a relatively flat band to be constructed by keeping a small set of hopping amplitudes. The relevant flatness parameter, which should be large for the effects of interactions to be important, is the ratio of the band gap (which sets a bound on the strength of the interactions one can safely include) to the bandwidth and they show how to get this number up to seven with just second-neighbor interactions. Similarly, Tang *et al.*[1], and Sun *et al.*[3] construct models on the kagome and checker-board lattices, which also exhibit large values of the flatness parameter.

With a flat Chern band in hand, Neupert *et al.* introduce interactions and study the system at a fractional filling of 1/3 through numerical computation on a modestly sized system. They find two of the classic signatures of the 1/3 FQHE state: a fractional quantum Hall conductance that was close to the filling fraction, and a nontrivial ground-state degeneracy with periodic boundary conditions. As a test, they vary the band structure continuously to a topologically trivial band and find that these features go away. In a related piece of unpublished work, another group finds similar results at fillings of 1/3 and 1/5[6]. Altogether, this work offers strong evidence that fractionally filled Chern bands do indeed exhibit the FQHE.

This is perhaps a good place to note that on a lattice the distinction between having a net magnetic field and not having it at all is not as sharp as it may seem. Essentially, it is always possible to stick a full flux quantum through some subset of loops on the lattice to shift the average magnetic field without affecting the actual physics. From this perspective, the physics in these flat-band models has a family resemblance to earlier studies of lattice versions of the FQHE [7, 8] with uniform magnetic fields. In this earlier work, the authors studied a fixed filling factor while varying the flux per plaquette from small values and large unit cells, where the standard Landau level description holds, to somewhat larger flux values and smaller unit cells, where that description broke down. As they were able to change this parameter without any evidence of encountering a phase transition, the latter limit constituted an observation of the FQHE in the presence of strong lattice effects. Needless to say, a more analytic approach can be expected to clarify this possible equivalence between this earlier "Hofstadter" and the current "Haldane" versions of the FQHE.

The present work also leaves open several other interesting questions: Can analytic expressions for the wave functions for the ground states and elementary excitations of the FQHE in the lattice models be found, and can they be related to those for the continuum FQHE? Can these FQHE states be realized in materials via this route at high temperatures, as speculated by Tang *et al.*[1]? Further afield, while an experimental example of a Chern insulator (an insulator with filled bands with a nonzero net Chern number, such as the Haldane model) has yet to be found, at least one example of the related two-dimensional topological insulators with time-reversal symmetry has been found. The band structures in such topological insulators also have nontrivial topology and

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can even be understood in terms of Chern numbers in certain cases. This raises the possibility that a time-reversal-symmetric version of the FQHE may arise in such models when interactions are added, and may be more readily observable. More ambitiously, it invites the conjecture that three-dimensional "strong" topological insulators, whose band topology is now qualitatively different from that denoted by the Chern numbers that are the focus of attention in this current work, might also give rise to fractional analogs by the same mechanism of including interactions in flattened bands with nontrivial topology.

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