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Viewpoint

The dialogue between quantum light and matter

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Although the quantum Rabi model is the simplest model describing the coupling of quantum light and matter, only now has an analytical solution been found.

Subject Areas: **Optics, Quantum Physics**

A Viewpoint on:
Integrability of the Rabi Model
D. Braak
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The Rabi model (RM) describes the simplest interaction between light and matter. In its semiclassical form, this model describes the coupling of a two-level system and a classical monochromatic field. The fully quantum model considers the same situation, with the light field quantized. Although this model has had an impressive impact on many fields of physics [1]—in its semiclassical form, it is the basis for understanding nuclear magnetic resonance—many physicists may be surprised to know that the quantum Rabi model has never been solved exactly. In other words, it has not been possible to write a closed-form, analytical solution for it. Now, thanks to a paper appearing in *Physical Review Letters* by Daniel Braak at the University of Augsburg in Germany, this model may be declared solved [2]. As physicists gain intuition for Braak's mathematical solution, the result could have implications for further theoretical and experimental work that explores the interaction between light and matter, from weak to extremely strong interactions.

Originally, the Rabi model was intended to describe the effect of a rapidly varying, weak magnetic field on an oriented atom possessing nuclear spin [3]. It was applied with success to explain the challenging experimental data taken by R. Frisch and E. Segrè [4]. Nowadays, it is applied to a wide variety of physical systems in quantum optics or condensed matter that enjoy similar dynamics. A few examples include microwave and optical cavity quantum electrodynamics (QED), atomic physics, quantum dots, trapped ions, superconducting qubits (artificial atoms), and circuit QED. The quantum Rabi model is a building block for the implementation of diverse protocols in contemporary quantum information [5], with potential applications to future quantum technologies.

The quantum Rabi model is simple in form: The coupling between light and matter is a dipole interaction; the matter (typically an atom) is assumed to have only two discrete states, and only a single mode of the quantized field participates in the interaction (Fig. 1). In most cases, the model is further simplified by making the rotating-wave approximation (RWA), where coupling terms that simultaneously excite or de-excite the atom and the field are neglected. Since most experiments use microwave or visible light, this approximation is justified because the coupling strength between the atom and the field is several orders of magnitude smaller than the frequencies of the involved parts. There are, nevertheless, some counterintuitive physical effects beyond the rotating wave approximation. For example, the dropped terms can lead to a noticeable shift in the resonance frequency (the Bloch-Siegert shift) and the interplay of chaotic regimes, among others.

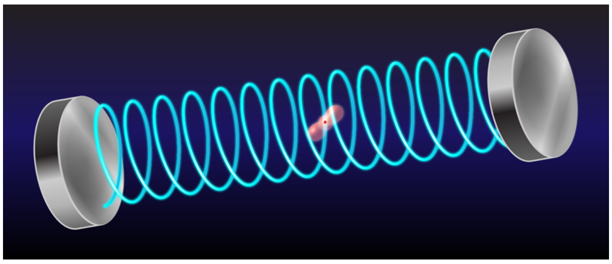

FIG. 1: The Rabi model describes the simplest interaction between quantum light and matter. The model considers a two-level atom coupled to a quantized, single-mode harmonic oscillator (in the case of light, this could be a photon in a cavity, as depicted in the figure). The model applies to a variety of physical systems, including cavity quantum electrodynamics, the interaction between light and trapped ions or quantum dots, and the interaction between microwaves and superconducting qubits. (APS/Alan Stonebraker)

In the quantum version of the Rabi model, the application of the rotating wave approximation is known as the Jaynes-Cummings model. The latter served as a theoretical and experimental milestone in the history of

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quantum physics. Its dynamics is integrable—meaning the time dependence of most of the interesting properties can be described by explicit analytical expressions—and splits the space of quantum states in an infinite sequence of state doublets. However, the reason the Jaynes-Cummings model has been so successful is not merely because it can be solved; it accurately predicts a wide range of experiments. Analytical solutions of this model have brought clarity and intuition to several important problems and experimental results in modern physics.

Braak's unexpected full analytical solutions of the quantum Rabi model are, however, worthy of celebration [2]. In mathematics and physics there are many, not necessarily compatible, criteria for a model to be both integrable and solvable. Examples include Frobenius' condition for integrability in differential systems and Liouville's condition for integrability in dynamical systems [6]. In the realm of quantum physics, it has been assumed that the existence of invariant subspaces associated with conserved quantities, other than energy, might be a necessary condition, as is the case in the Jaynes-Cummings model. The quantum Rabi model possesses naturally an additional discrete symmetry, the parity, which was assumed for some time to be insufficient for yielding a solvable model. Braak has proved that this is not the case and has presented exact analytical solutions of the quantum Rabi model for all parameter regimes. This is a remarkable achievement that adds the model to the short list of integrable quantum systems. Furthermore, Braak is able to take advantage of this key result to propose an operational criterion of integrability, inspired by the case of the hydrogen atom.

Integrability is, following Braak, equivalent to the existence of quantum numbers that classify eigenstates uniquely. It does not presuppose the existence of a family of commuting operators. Surprisingly, he has been able to apply this novel integrability criterion to a more elaborate case, the generalized quantum Rabi model, where a term that allows tunneling between the two atomic states is added. For the latter, he was able to prove that the model is not integrable, because it has an additional symmetry that is broken. However, the model is exactly solvable, and Braak presents the solutions. These are important results advancing the mathematical aspects of the quantum Rabi model in terms of integrability and solvability. Moreover, we should not overlook that the quantum Rabi model is a key physical model describing the interaction of quantum light and matter.

We might have been misled to think that, after all, it is the Jaynes-Cummings and not the quantum Rabi model that has a physical relevance in present experiments. Recent achievements in circuit QED prove that this is not the case [7]. They have allowed us to access the ultrastrong coupling regime of light-matter interactions [8], where the coupling strength is large with respect to the relevant frequencies. For such a system, only the full quantum Rabi model can describe the observed physics. Furthermore, in the near future, it is expected that the experiments could reach the deep strong coupling regime [9], where the ratio of the coupling strength to the relevant frequencies exceeds unity. In this case, it is crucial to abandon approaches involving perturbative methods and the concept of Rabi oscillations—the well-known oscillations between atomic state doublets predicted by the Jaynes-Cummings model [1]. Rather, one will be able to describe novel physical features, such as parity chains and photon number wave packets [9].

Ultimately, we can expect that the exact solutions of the quantum Rabi model will be relevant for cutting-edge experiments that can access all kinds of light-matter coupling regimes. It will be interesting to study how the proposed integrability criterion applies and influences the physics of multipartite atom-field systems, dissipative models, and even the quantum simulation of the quantum Rabi model in setups that are usually described by the Jaynes-Cummings model [10]. An intense dialogue between mathematics and physics will be needed to describe and predict unprecedented physical phenomena, since they might be hidden in the quantum numbers associated with Braak's integrability criterion and analytical expressions. Otherwise, the present achievement will remain but a mathematical monologue.

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