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Viewpoint

Taming Nonlinear Freak Waves

Holger Hennig
Department of Physics, Harvard University, Cambridge, MA 02138, USA and
Broad Institute of MIT and Harvard, Cambridge, MA 02142, USA

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Nonlinear freak waves in water can be generated experimentally by exploiting the time-reversal symmetry of the equations that govern their propagation.

Subject Areas: **Nonlinear Dynamics, Fluid Dynamics**

A Viewpoint on:
Time-Reversal Generation of Rogue Waves
Amin Chabchoub and Mathias Fink
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Freak or rogue waves are giant waves in the ocean, up to tens of meters high, that seem to form out of the blue, posing a major threat for even the largest ships. It is no wonder that freak waves have often been described as a mysterious phenomenon. How can they form and why do they occur orders of magnitude more often than what is expected from commonly used models, such as a “random-plane-wave model” based on the linear superposition of plane waves? One possible mechanism for their formation is via so-called “breathers,” oscillating localized and high-amplitude waves that can occur in nonlinear media.

Breathers are solutions of the nonlinear Schrödinger equation (NLS), which describes the dynamics of a large variety of nonlinear media including water, optical fibers, and clouds of Bose-Einstein condensates. As reported in *Physical Review Letters*, Amin Chabchoub at the Swinburne University of Technology in Melbourne, Australia, and Mathias Fink at the Institut Langevin of the French National Center for Scientific Research (CNRS) have demonstrated the experimental generation of breathers in water [1] based on a particular mathematical property of the NLS: its time-reversal symmetry. This property allows the authors, for the first time, to “refocus” breathers: after they are generated, propagated, and are recorded at some distance from the origin, the decayed nonlinear wave profile is time-reversed and re-emitted in such a way that their energy focuses back, as though a movie of the propagating waves had been played backwards. The effect, which occurs despite the inevitable presence of damping and noise, may be exploited for the generation of breathers in a variety of media or to study the role of nonlinearity in the formation of rogue waves.

Historically, the simplest approach to model freak waves has been the random-plane-wave model, in which the linear superposition of waves leads to a statistical (Rayleigh) distribution of wave intensities. Yet, as confirmed by recent research on freak waves in microwave resonators [2], it has been known for many years that this underestimates by orders of magnitude the probability for the occurrence of freak waves. This puzzle remains unsolved even when the model is refined by including multiple wave-scattering events. But two other physical effects could be responsible for the unexpectedly high frequency of freak waves observed in the real world: (i) nonlinearity, i.e., the formation of breathers and (ii) caustics, i.e., the focusing of waves to high intensity due to purely linear wave propagation in random media.

Caustics form naturally in random media and act like lenses for waves. They determine, for instance, the changing light patterns that you can see on the bottom of a swimming pool on sunny days. While the contribution of caustics to freak waves has recently been uncovered [3], the role of breathers in the formation of freak waves in the ocean is still foggy. Therefore, a way to experimentally control freak waves formed via breathers would be highly desirable.

Chabchoub and Fink tackle the nonlinear contribution to freak wave dynamics experimentally. They use a 15-meter-long unidirectional water flume with a water depth of 1 meter (m)—basically a water tank carefully optimized to control wave propagation in one dimension. To minimize dissipation and its influence on breather propagation in the tank, they cleaned the walls of the flume and used filtered water before performing the experiments. The researchers refocused breathers using the following protocol: (i) Building on previous work [4], they provided the initial conditions to generate a breather by moving a single wave-forming paddle. Such initial conditions, e.g., the amplitude profile of the breather, can be calculated analytically. (ii) They measured the amplitude profile of the decaying breather at a 9-m distance. (iii) They calculated and generated the time-reversed amplitude profile of the breather.

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The time-reversal symmetry of the NLS would imply that a state at a given time could be back-propagated to yield the initial conditions that generated it. The time-reversed signal would then lead to the re-formation of a breather after propagating for 9 m. But proving that the mathematically predicted refocusing could work as planned under real-world experimental conditions was the greatest challenge for the researchers. Refocusing has been demonstrated in the linear regime for single giant peaks, also called hot spots [5]. Can nonlinearity-based refocusing also succeed, despite the inevitable damping that breaks the time-reversal symmetry of the NLS? Chabchoub and Fink indeed report a successful outcome: the experimental refocusing of a first-order (so-called Peregrine) breather. Furthermore, they were able to reverse in time an even higher (second) order (Akhmediev-Peregrine) breather. The amplitude of the wave envelope is amplified by a factor of 5 in the case of the second-order breather. Thus, if a ship were to sail across 2-m random waves in the ocean, encountering a second-order breather would translate to facing waves up to 10 m high.

It is worth noting that time reversal of breathers as proposed by the authors may provide insight into a broader class of situations beyond freak waves. Moving breathers (more widely known as solitons) manifest themselves in rivers as part of tidal bores, in the form of a wave front followed by a train of solitons [6] (see Fig. 1). This phenomenon is well known among surfers: the tidal bore in the Severn River in the UK can carry surfers several miles upstream. A five star bore—the highest category—attracted surfers and “breather watching” spectators alike in February 2014.

The authors’ results may also have implications in other domains of physics. The NLS describes a variety of physical systems, including Bose-Einstein condensates (BECs) confined to cigar-shaped traps—a system that can be controlled experimentally with a very high precision. In the presence of an optical lattice potential, the NLS turns into a lattice equation: the discrete NLS. Generating breathers in BECs is appealing, as breathers are stable and localized matter waves that are coherent on long time scales [7], properties that could be used, for instance, in quantum computing. Experiments with BECs may address whether time reversal of decayed breathers is possible. However, quantum effects may pose an obstacle: classically instable motion (including the decay of a breather) leads to decoherence [8], which goes beyond the validity of the NLS.

While the scheme studied by the authors is one dimensional (like solitons in narrow rivers), future research may investigate the simulation of breathers in two dimensions, providing a more realistic approach to understand possible mechanisms for the formation and decay of freak waves on the ocean surface. Furthermore, disentangling nonlinear contributions (i.e., breathers) from linear contributions (i.e., caustics) to freak-wave formation remains an outstanding challenge for researchers in a wide range of fields, including microwave chaos, nonlinear optics, and water waves.

FIG. 1: Both linear and nonlinear interactions in water can lead to the formation of high-intensity waves (so-called freak or rogue waves). Similar nonlinear mechanisms are responsible for the generation of solitons in rivers, such as those traveling upstream on the Severn River during tidal bores, which attract river surfing enthusiasts (see video coverage of the Severn bore at http://www.youtube.com/watch?v=IKA39LQOIck). (www.thesevernbore.co.uk)

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About the Author

Holger Hennig

Holger Hennig is a Postdoctoral Fellow at the Broad Institute and the Department of Physics at Harvard University in Cambridge, MA. Previously, he was at the Max Planck Institute for Dynamics and Self-Organization in Göttingen, Germany, where he received a Ph.D. in physics. He uses concepts and tools from nonlinear dynamics and statistics to study complex systems. His research interests include the dynamics of ultracold atoms in optical lattices with a focus on breather formation and the study of human musical rhythms. For more information see http://www.nld.ds.mpg.de/ holgerh.