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Viewpoint

An Equation of State for Active Matter

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An equation of state for a gas of self-propelled spheres is a step towards a thermodynamic description of "active" matter, such as bird flocks and tissue.

Subject Areas: **Statistical Physics**

A Viewpoint on:
Pressure and Phase Equilibria in Interacting Active Brownian Spheres
Alexandre P. Solon, Joakim Stenhammar, Raphael Wittkowski, Mehran Kardar, Yariv Kafri, Michael E. Cates, and Julien Tailleur
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Bird flocks, mammal herds, cellular tissue, and artificial self-propelled colloids are all examples of "active" matter—an assembly of self-driven units that take energy from the environment to produce motion [1]. Active matter is, in many ways, analogous to a gas of atoms or molecules, except that the particles are far from equilibrium, and therefore cannot be described with standard statistical physics. A thermodynamic equation of state, such as the relation between pressure and density in an ideal gas, would, however, be useful to characterize these systems. Such an equation could, in the long term, lead to a "theory" for certain forms of living matter and the principles that underlie their dynamics. Taking an important step towards this goal, Alexandre Solon at Diderot University in Paris and his colleagues have derived an equation of state for a "gas" of self-propelled colloidal spheres. The equation relates the pressure the spheres exert on a wall to the spheres' properties, and the predictions of this equations are potentially testable in, for example, solutions of motile colloids [2].

Several characteristics distinguish active matter from more studied far-from-equilibrium systems. First, every particle in an active matter system is out of equilibrium. This is in contrast to "boundary driven" systems, like a strip of metal heated at one end, that are locally equilibrated. Second, active systems can exist in homogeneous states—again at odds with boundary driven systems, which, by design, vary from point to point. And since homogeneity is a basic assumption of standard thermodynamics, active matter systems are potentially amenable to a thermodynamic description. Finally, active matter can exhibit cooperative phenomena, like collective motion [3, 4].

Active matter systems are, however, difficult to describe with standard statistical physics methods. In equilibrium systems, energy fluctuations come from exchanges with a single heat bath, but in active matter, energy injection and dissipation occur by different mechanisms. As a result, thermodynamic quantities like pressure or temperature may be ill-defined. Solon *et al.* have, for example, shown in a separate work that the pressure exerted on a wall by active nonspherical particles is not an intrinsic property of the gas, and instead depends on the interaction between the particles and the wall [5].

In their new work, Solon *et al.* modeled active matter as a gas of spherical self-propelled particles (Fig. 1) moving in a viscous fluid [2]. They assume a strong viscous drag, so that the velocity of the particle is proportional to the total force acting on it. They also attach a "heading vector" to each particle, which defines the direction of the propelling force. The angle of this heading vector slowly rotates, but its direction of rotation is random. Finally, the authors added three interaction forces: a short-range force between particles, a repulsive force between particles and the wall, and a random force that accounts for collisions between the particles and the much smaller particles that make up the fluid.

The authors then derived an equation that gives the probability of finding a particle at a certain position with a certain heading vector. By integrating this equation along a line from the wall to a point within the fluid, the authors could express the mechanical pressure the particles exert on the wall in terms of only the particles' properties, independently of the properties of the wall. The pressure has three parts: an ideal pressure term, similar to the perfect gas equation of state; a "direct" contribution that depends on the interaction between the particles; and an "indirect" contribution, which depends on the coupling between self-propulsion and interaction forces between particles. This third term results from a slowing down of particle motion because of the presence of other particles, similar to the slowdown of cars

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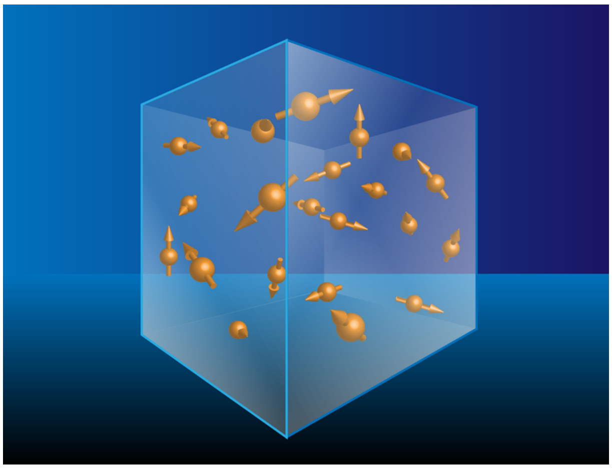

FIG. 1: Self-propelled particles serve as a model for active matter, a type of matter that includes tissue, groups of moving animals, and chemically active colloidal particles. The heading vector (arrow) on each particle indicates the particle's propulsion force, whose direction diffuses in time. Solon *et al.* derived an equation of state for the pressure the particles exert on a wall. As in an ideal gas, this pressure is uniquely defined by the bulk properties of the gas. (APS/Alan Stonebraker)

approaching a traffic jam. This contribution is the most interesting one, since it vanishes in the absence of self-propulsion and is thus a genuine nonequilibrium effect.

This model active matter system shares a similarity with equilibrium matter: It can form a phase-separated state with different densities, and the calculated pressures in these states have equal values. Yet, although the pressure calculated by Solon *et al.* seems to have the required properties of a thermodynamic quantity, the analogy is not complete: The densities of the two phases fail a standard thermodynamic test for the stability of two coexisting phases (the Maxwell construction). As such, we should not think of the pressure exerted by the active spheres as equivalent to a pressure in an equilibrated system.

In addition to its conceptual interest, the work by Solon *et al.* predicts the osmotic pressure that should be exerted on a container by actual active colloids in a solution. This quantity could potentially be compared to experiments, for example, with colloidal particles that start moving when exposed to light [6], or because they have two faces with different coatings [7]. A more accurate model should, however, account for the forces between particles that are mediated by the fluid (hydrodynamic interactions).

Physicists are far from having a full statistical physics framework that can describe active matter. It is unclear, for example, if it will be possible to define thermodynamic quantities like free energy, temperature, or chemical potential in a consistent way, or even if these quantities offer the most useful description of active matter. Recent progress describing active matter is reminiscent of the early days of statistical mechanics more than a century ago: Physicists derived the kinetic theory of gases first and only later connected these ideas to thermodynamics. Similarly, the recently developed kinetic theory of interacting self-propelled particles can predict collective motion and pattern formation, but it has not been related to thermodynamic notions. The work of Solon *et al.* and other researchers [8–10], paves the way to new approaches in the statistical physics of active matter that are more focused on a thermodynamic description.

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Eric Bertin studied theoretical statistical physics at the CEA in Saclay, and received his Ph.D. in 2003. After a postdoc at the University of Geneva, he was appointed as a CNRS Research Associate in the Physics Department of the Ecole Normale Supérieure de Lyon. In 2013, he moved to LIPHY, an interdisciplinary physics laboratory of the CNRS and Joseph Fourier University in Grenoble. His current research focuses on the statistical characterization of non-equilibrium physical systems and on applications of statistical physics concepts and methods to other fields, such as statistical signal processing and the modeling of social systems.

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