Measurement of the $2^+ \rightarrow 0^+$ ground-state transition in the $\beta$ decay of $^{20}$F

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We report the first detection of the second-forbidden, nonunique, $2^+ \rightarrow 0^+$, ground-state transition in the $\beta$ decay of $^{20}$F. A low-energy, mass-separated $^{20}$F$^+$ beam produced at the IGISOL facility in Jyväskylä, Finland, was implanted in a thin carbon foil and the $\beta$ spectrum measured using a magnetic transporter and a plastic-scintillator detector. The $\beta$-decay branching ratio inferred from the measurement is $b_\beta = [0.41 \pm 0.08$ (stat) $\pm 0.07$ (sys)] $\times 10^{-5}$ corresponding to $\log ft = 10.89(11)$, making this one of the strongest second-forbidden, nonunique $\beta$ transitions ever measured. The experimental result is supported by shell-model calculations and has significant implications for the final evolution of stars that develop degenerate oxygen-neon cores. Using the new experimental data, we argue that the astrophysical electron-capture rate on $^{20}$Ne is now known to within better than 25% at the relevant temperatures and densities.

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I. INTRODUCTION

Second-forbidden, nonunique $\beta$ transitions ($\Delta J = 2$, $\Delta \pi = 0$) typically have very small branching ratios, which makes their detection rather challenging. Only around 27 such transitions have been observed [1]. Measurements of the rates and shapes of forbidden $\beta$ transitions provide insights into nuclear structure and occasionally also into astrophysical processes.

Recent studies have highlighted the importance of the second-forbidden, nonunique, electron-capture transition from the $0^+$ ground state of $^{20}$Ne to the $2^+$ ground state of $^{20}$F for the final evolution of stars of 7–11 solar masses that develop degenerate oxygen-neon cores [2–4]. The strength of the transition is, however, not well constrained, neither experimentally nor theoretically, making an experimental determination highly desirable. The strength may be determined from the branching ratio of the inverse $2^+ \rightarrow 0^+$ transition in the $\beta$ decay of $^{20}$F (Fig. 1), but this transition is not easily detected as it is masked by the much faster, allowed, $2^+ \rightarrow 2^+$ transition to the first-excited state in $^{20}$Ne. Indeed, previous attempts to detect the $2^+ \rightarrow 0^+$ transition have been unsuccessful [7–10] yielding a rough upper limit of $\sim 10^{-5}$ on the branching ratio [10]. The $\beta$-decay endpoint energies for the ground-state and first-excited state transitions are 7.025 and 5.391 MeV, respectively, leaving a rather narrow energy window for the detection of the ground-state transition. Here, we report the first successful measurement of the second-forbidden, nonunique, $2^+ \rightarrow 0^+$ transition in the $\beta$ decay of $^{20}$F, present shell-model calculations which corroborate the experimental result, and determine the impact on the

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†The endpoint energies are known to sub-keV precision [5,36].
The experiment was performed at the IGISOL-4 facility of the JYFL Accelerator Laboratory in Jyväskylä, Finland [12,13]. Radioactive ions of $^{12}$B$^+$ and $^{20}$F$^+$ were produced via $(d, p)$ reactions on targets of boron (B) and barium fluoride (BaF$_2$). The K130 cyclotron was used to produce the deuterium beam, which had an energy of 9 MeV and an intensity of around 10 $\mu$A. For the production of the $^{20}$F$^+$ ions, a 53 $\mu$m thick tantalum (Ta) degrader was used to reduce the beam energy to 6 MeV. The B and BaF$_2$ targets were 0.5 mg/cm$^2$ and 1.2 mg/cm$^2$ thick with backings of 4.5 $\mu$m tantalum (Ta) and 2 $\mu$m tungsten (W), respectively. The reaction products were thermalized in the IGISOL ion guide gas cell, using helium at a pressure of around 100 mbar for $^{12}$B and 250 mbar for $^{20}$F, and extracted with a sextupole ion guide [14]. After acceleration to 30 keV, the ions were separated based on their mass-to-charge ratio using a dipole magnet, before being guided to the experimental station where they were stopped in a thin (50 $\mu$g/cm$^2$) carbon foil. The $\beta$ spectrum of $^{20}$F was the primary interest of the experiment while the $\beta$ spectrum of $^{15}$B provided important calibration data.

The detection system, shown in Fig. 2, consisted of a Siegbahn-Slätis type intermediate-image magnetic electron transporter [15] combined with an energy-dispersive detector. Such an arrangement is well suited for the measurement of rare ground-state transitions in nuclear $\beta$ decays as the effective solid angle of the detector, and hence the count rate, is greatly increased by the focusing action of the magnetic field. Furthermore, and equally important, the shield on the center axis prevents $\gamma$ rays and electrons produced by transitions to excited states in the daughter nucleus from reaching the detector. This essentially eliminates $\beta\gamma$ summing and $\beta\beta$ pile-up as sources of background and leads to an improved sensitivity towards the ground-state transition.

The magnetic transporter was constructed at the Department of Physics, University of Jyväskylä (JYFL) in the 1980s [16], but has been fully refurbished for the present experiment. The $\beta$ detector, shown in Fig. 3, has the shape of a cylinder and consists of a 5-mm-thick outer detector, used as a veto against cosmic rays, and a 45 $\times$ 45 mm$^2$ inner detector, used to measure the full energy of the electrons. The inner detector is further subdivided into a 5-mm-thick front detector and a 40-mm-thick main detector to provide additional discriminative power. All three detectors (veto, front, and main) are plastic scintillators read out with silicon photomultipliers. The detector dimensions represent a compromise between the requirement to fully stop a significant fraction of the most
energetic electrons (the nominal range of 7-MeV electrons in plastic is 35 mm) and the requirement to minimize the cosmic-ray exposure.

Some of the calibration data presented in this paper were obtained using an earlier, two-channel version of the three-channel detector that we have just described. The two detectors have identical dimensions and only differ in one respect: the inner volume of the two-channel detector is not divided into a front and a main section. When necessary we use the labels v1 and v2 to distinguish between the two-channel (v1) and three-channel (v2) detector. When no label is provided it is understood that the three-channel detector was used.

A small LaBr3(Ce) crystal placed inside the shield on the center axis was used to detect the 1.63-MeV γ-ray from the decay of 208F, thereby providing absolute normalization of the β spectrum. Finally, a baffle placed at the center of the magnet prevented positrons, which spiral in the opposite direction of electrons, from reaching the detector thereby eliminating positron emitters as a potential source of background, while reducing the electron flux by only 11%.

III. DATA ANALYSIS AND RESULTS

A. Characterization of the β response

By only allowing electrons within a relatively narrow energy band to reach the magnetic transporter effectively “carves out” a slice of the β spectrum. This is demonstrated in Fig. 4, which shows energy spectra measured at three different magnetic-field strengths. The central energy selected by the magnetic transporter is approximately given by \( \tilde{E}_\beta \approx 7.72I + 3.01F \) MeV, where \( I \) is the electrical current expressed as a fraction of the maximum current provided by the power supply (700 A). The spectra obtained at \( \tilde{I} = 35.3\% \) and 56.4% show a central slice and the upper end of the allowed β spectrum of 208F, respectively. The spectra obtained at 67.0% and 79.0% show slices of the β spectrum of 12B, which has an end-point energy of 13.37 MeV. In all cases, the main peak is well reproduced by the GEANT4 simulation. Deviations occur in the low-energy tails, especially for the spectra obtained at the higher current settings, but these deviations are not important for the present analysis. The 208F spectra have been cleaned by requiring that no coincident signal is recorded in the veto detector (veto cut) and that the energy deposited in the front detector is between 0.65–1.60 MeV (front cut). The 12B spectra have also been subject to the veto cut, but the front cut could not be applied to these spectra because the 12B measurements were performed with the two-channel β detector.

The 208F and 12B data (only a subset of which are shown in Fig. 4) and data obtained with a calibrated 207Bi source, have been used to validate the absolute accuracy of the GEANT4 simulations all the way up to \( \tilde{E}_\beta = 8.0 \) MeV. As shown in Fig. 5, the simulated and experimental β transmission efficiencies exhibit reasonable agreement across the full energy range, with the simulation overestimating the transmission efficiency by 8% on average. The transmission efficiency is determined as \( \varepsilon_\beta = N_p N_\gamma^{-1} \varepsilon_\gamma \), where \( N_p \) is the number of counts in the full-energy peak in the uncleaned β spectrum (i.e., before application of the veto and front cuts), \( N_\gamma \) is the number of 1.63-MeV γ rays, and \( \varepsilon_\gamma \) is the γ-ray detection efficiency, cf. Sec. III D. We note that the overall normalization of the 12B data points could not be established experimentally due to the lack of a sufficiently intense γ-ray line. However, by monitoring the β count rate we were able to establish that the implantation rate was constant throughout the measurements, implying that the data points share the same overall normalization; its value was determined via a \( \chi^2 \) fit to the data. The 8% overestimation may partly or entirely be attributed to the uncertainty on the γ-ray detection efficiency, which causes a 5% uncertainty on the normalization of the experimental transmission efficiency, cf. Sec. III D.

FIG. 4. Comparison of experimental and simulated energy spectra obtained at 35.3%, 56.4% (208F), 67.7%, and 79.0% (12B) of the maximum electrical current. The 208F data have been subject to both veto and front cut, while the 12B data have been subject only to the veto cut.

FIG. 5. Ratio of simulated and experimental transmission efficiencies, \( \varepsilon_\beta^{(\text{sim})}/\varepsilon_\beta^{(\text{exp})} \), versus the central energy selected by the magnetic transporter, \( \tilde{E}_\beta \). The data are labeled by the isotope (12B, 208F, 207Bi) and the detector (v1, v2) used for the measurement. The gray band indicates the systematic uncertainty on the normalization of the 208F data. The dashed line was obtained from a \( \chi^2 \) fit to the data in which the normalization of the 12B data was allowed to vary freely (see text for details).
The large scatter in the experimental data points observed in Fig. 5 may be attributed to temporal variations in beam optics, which affect the source geometry and hence the transmission efficiency. The occurrence of such temporal variations is evident in Fig. 6, which shows the transmission efficiency obtained in nine separate runs performed at the same magnetic-field strength ($I = 35.3\%$) at different times during the experiment. The temporal variations amount to a 13% spread in transmission efficiency, which we include as a systematic uncertainty on the final result.

Finally, we examine the cut acceptance, $\eta$, defined as the fraction of counts in the full-energy peak that survive the veto and front cuts. As shown in Fig. 7, the simulation tends to overestimate the cut acceptance, partly due to the presence of cross-talk between the inner and the outer detectors, but also due to inaccuracies in the modeling of the stopping process in the detector volumes. The factor by which the simulation overestimates the cut acceptance is small for $E_\beta < 3.0$ MeV, but grows with increasing energy reaching $1.25(9)$ at $E_\beta \sim 6.0$ MeV.

In Table I we summarize the sources of systematic uncertainty affecting the normalization of the $\beta$ spectrum. In each case we give the estimated correction factor to the normalization of the GEANT4 simulation at $E_\beta \sim 6.0$ MeV and the estimated uncertainty.

B. Long-duration measurements

Long-duration measurements were performed at the current settings $I = 67.7\%$ (67 h) and 70.5% (38 h) to search for a signal in the energy region 5.4–7.0 MeV, and at 79.0% (37 h) to demonstrate that any signal detected at the two lower settings did not persist above 7.0 MeV. The average $^{20}$F implantation rate for these measurements was 11 kHz, while the $\gamma$ and $\beta$ count rates were at most a few tens of Hz and a few Hz, respectively, implying negligible dead time. Additionally, background data were collected at 67.7% and 70.5% for a total of 183 h. The $\beta$ spectrum obtained at 67.7% is shown in Fig. 8. The cosmic-ray background dominates

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**Table I. Sources of systematic uncertainty in the normalization of the experimental $\beta$ spectrum.** In each case, we give the correction factor by which the spectrum has been multiplied (second column) and the uncertainty on this factor (third column). The total uncertainty was obtained by adding the individual contributions in quadrature.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$\gamma$-ray detection efficiency</td>
<td>1</td>
<td>5%</td>
</tr>
<tr>
<td>$\beta$ transmission efficiency</td>
<td>$1/1.08$</td>
<td>13%</td>
</tr>
<tr>
<td>Cut acceptance</td>
<td>$1/1.25$</td>
<td>7%</td>
</tr>
<tr>
<td>Total</td>
<td>$1/1.35$</td>
<td>16%</td>
</tr>
</tbody>
</table>

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**FIG. 6.** $\beta$ transmission efficiency obtained in nine separate runs performed at a magnetic-field strength of $I = 35.3\%$ at different times during the experiment. The gray band indicates the spread attributed to variations in beam optics.

**FIG. 7.** Ratio of simulated and experimental cut acceptances, $\eta^{\text{sim}}/\eta^{\text{exp}}$, versus the central energy selected by the magnetic transporter, $E_\beta$. The data are labeled by the isotope ($^{12}$B, $^{20}$F, $^{207}$Bi) and the detector (v1, v2) used for the measurement, and the cuts applied in the data analysis (veto and front). The solid lines and the hatched area show trend lines and the estimated uncertainty, respectively.

**FIG. 8.** $\beta$ spectrum with cuts (gray) and without (yellow) obtained in the inner plastic-scintillator detector at 67.7% of the maximum electrical current. The dashed box indicates the region used for the likelihood fits shown in Fig. 11.
the raw $\beta$ spectrum above 5.4 MeV while electrons from the allowed $2^+ \to 2^+$ transition produce the bump centered at 5.0 MeV and the continuum below it. In the signal region the cosmic-ray background is reduced by a factor of 100 by the veto cut. The front cut removes another factor of 3.5 resulting in an overall reduction factor of 350. Meanwhile, about 2/3 of the $\beta$ particles survive the cuts, a fraction which would have been even larger in the absence of cross-talk between the inner and outer detectors.

C. Investigation of the residual background

Below 3 MeV, $\gamma$ rays, chiefly from the decays of $^{40}$K and $^{208}$Tl, are the main source of background, while cosmic-ray muons dominate above this energy, resulting in a background rate of 150 counts/h in the signal region (5.8–6.8 MeV). Measurements performed at different times of the year and different locations within the laboratory verified that this rate was very nearly constant to within a few percent. The residual background rate in the signal region after application of the veto cut was 1–2 counts/h. The energy dependence of the residual background is markedly different from the energy dependence of the raw background, indicating a different physical origin. To further characterize the residual background, a background measurement was performed at Callio Lab in the Pyhäslami mine in Pyhäjärvi, Finland, at the depth of 1430 m (4100 m.w.e.) where the cosmic-ray muon flux is greatly suppressed [17]. In Fig. 9 we compare the spectrum obtained underground to a spectrum obtained at the surface. This comparison clearly demonstrates that the residual background is cosmic-ray induced. We note that placing the detector inside the magnetic transporter had little or no effect on the residual background. Similarly, the magnetic field seemed to exert little or no influence on the residual background. We therefore conclude that hadronic secondaries, and neutrons in particular, from cosmic-ray interactions in the atmosphere and the roof of the laboratory are the likely source of the residual background. Finally, we note that the likely source of the residual background.

D. Absolute normalization

Returning to the long-duration $^{20}$F measurements, we show in Fig. 10 the $\gamma$ spectrum obtained at 67.7%. The well-resolved 1.63-MeV line from the decay of $^{20}$F, which is used for absolute normalization, is clearly visible. The efficiency of the LaBr$_3$(Ce) $\gamma$-ray detector at 1.63 MeV was determined online from the ratio of $\beta\gamma$ coincidences and $\beta$ singles events, yielding the value $\varepsilon_{\gamma} = 0.59(3) \times 10^{-4}$. This online calibration was confirmed by an offline calibration made using radioactive sources of $^{137}$Cs, $^{207}$Bi, $^{152}$Eu, and $^{60}$Co of known activities, which exhibit $\gamma$-ray lines with known relative intensities covering the energy range 0.3–1.8 MeV.

E. Detection of the forbidden transition

Figure 11 shows the cleaned $\beta$ spectra obtained in the long-duration measurements, zoomed in on the region of interest. The spectra obtained at 67.7% and 70.5% reveal a clear excess of counts in the signal region when compared to the background spectrum. For example, the spectrum at...
FIG. 11. Likelihood fits to the experimental data in the energy region 5.0–8.0 MeV obtained with beam (a–c) and without (d), at current settings focused on the region of interest (67.7% and 70.5%) and immediately above it (79.0%). The contribution of the second-forbidden, ground-state transition in the $\beta$ decay of $^{20}$F is shown by the thin, solid (blue) curve.

67.7% has 55 ± 7 counts between 5.8–6.8 MeV while the background spectrum, downscaled to account for the longer measurement time, only has 30 ± 3 counts in the same region. Equally important, no excess of counts is observed above the signal region in the data obtained at 79.0%. Based on the measurements performed at lower current settings (Fig. 8) we can exclude $\beta\gamma$ summing and $\beta\beta$ pile-up as possible explanations. Furthermore, the analysis of the temporal distribution of the counts between 5.8–6.8 MeV shown in Fig. 12 reveals a clear correlation with the $^{20}$F implantation rate, which varied by more than a factor of two during the experiment, while the temporal distribution of the counts above 7.0 MeV shows no such correlation. Thus, the observed signal is consistent with being due to the hitherto unobserved, second-forbidden, ground-state transition in the $\beta$ decay of $^{20}$F.

FIG. 12. (a) Temporal variations in $\gamma$-ray count rate during 67 hours of measurement at $I = 67.7\%$. (b) Correlation between $\gamma$-ray count rate and $\beta$ count rate in the energy region $E_\beta = 5.8–6.8$ MeV. (c) Same as (b) but for the energy region $E_\beta > 7.0$ MeV.

IV. BRANCHING RATIO

The previous section has provided evidence that the second forbidden transition connecting the ground states of $^{20}$F and $^{20}$Ne has been measured. To convert the signal observed between 5.8–6.8 MeV into a branching ratio and hence determine the magnitude of the matrix element, we must estimate the unobserved part of the forbidden $\beta$ spectrum below 5.8 MeV, where the decay is dominated by the allowed transition. We do so guided by shell-model calculations as described in the following.

A. Shell-model calculations

For the calculation of the second-forbidden $\beta$-decay rate and $\beta$ spectrum we follow the formalism of Behrens and Bühring [18]. This formalism accounts for the distortion of the electron wave function due to the nuclear charge, which leads to the appearance of additional matrix elements when compared to the formalism of Walecka [19,20] used in Ref. [21].

The second-forbidden $\beta$-decay rate between the ground states of $^{20}$F and $^{20}$Ne can be expressed as

$$\lambda^\beta = \frac{\ln 2}{K} \int_1^q C(w)wp(q - w)^2F(Z, w)dw,$$

where $w = (E_\beta + m_e c^2)/m_e c^2$ is the total electron energy in units of $m_e c^2$, $p = \sqrt{w^2 - 1}$ is the electron momentum in
TABLE II. Matrix elements determining the shape factor of the second-forbidden transition between the ground states of $^{20}\text{F}$ and $^{20}\text{Ne}$. The second column shows the values obtained from a shell-model (SM) calculation with the USDB interaction. The third column shows the values obtained from a shell-model calculation in which the matrix elements have been constrained based on the conserved vector current (CVC) theory and the E2 strength of the decay of the isobaric analog state of $^{20}\text{F}$ to the ground state of $^{20}\text{Ne}$. The quenched matrix elements can be obtained by multiplying the axial values by a factor $1/g_A$. (See text for additional information.)

<table>
<thead>
<tr>
<th>Matrix element</th>
<th>SM</th>
<th>SM+CVC+E2</th>
</tr>
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<tbody>
<tr>
<td>$\langle F^0_{211} \rangle$</td>
<td>0</td>
<td>$-0.0118^a$</td>
</tr>
<tr>
<td>$\langle F^0_{220} \rangle$</td>
<td>0.252</td>
<td>0.184$^b$</td>
</tr>
<tr>
<td>$\langle F^0_{220}(1,1,1,1) \rangle$</td>
<td>0.301</td>
<td>0.229</td>
</tr>
<tr>
<td>$\langle F^0_{220}(2,1,1,1) \rangle$</td>
<td>0.287</td>
<td>0.210$^b$</td>
</tr>
<tr>
<td>$\langle F^0_{211} \rangle$</td>
<td>$-0.122$</td>
<td>$-0.122$</td>
</tr>
<tr>
<td>$\langle F^0_{211}(1,1,1,1) \rangle$</td>
<td>$-0.142$</td>
<td>$-0.142$</td>
</tr>
<tr>
<td>$\langle F^0_{211}(2,1,1,1) \rangle$</td>
<td>$-0.135$</td>
<td>$-0.135$</td>
</tr>
</tbody>
</table>

$^a$Matrix elements constrained from experimental data.

units of $m_ec$ and $g = (M_p c^2 - M_d c^2)/(m_ec^2)$ is the energy difference between the initial and final nuclear state, $M_p$ and $M_d$ being the nuclear mass of the parent and daughter nucleus. The constant $K = 6144 \pm 2$ s has been determined from superallowed Fermi transitions [22]. $F(Z, w)$ is the Fermi function, which arises due to the Coloumb interaction between the electron and the daughter nucleus with atomic number $Z$. Finally, $C(w)$ is the shape factor, which depends on the matrix elements of the transition. For the $2^+ \rightarrow 0^+$ second-forbidden nonunique transition the shape factor has the form

$$C(w) = a_0 + \frac{a_1}{w} + a_1 w + a_2 w^2 + a_3 w^3 + a_4 w^4,$$

with the coefficients $a_n$ given by combinations of seven matrix elements [18,23].

We have performed shell-model calculations in the sd-shell valence space using the USDB interaction [24] and the code NUSHELLX [25]. For the evaluation of the many-body matrix elements we use the single-particle matrix element expressions provided in Ref. [18] modified to account for the fact that our shell-model calculations follow the Condon-Shortley phase convention [26] instead of the Biedenharn-Rose phase convention [27]. The resulting matrix elements are shown in Table II. Our calculations use harmonic-oscillator single-particle wave functions with a radial parameter of $b = 1.86$ fm and a uniform charge radius of $R = 3.88$ fm. These values reproduce the root-mean-square radius of $^{20}\text{Ne}$ determined from x-ray spectroscopy of muonic atoms [28]. Using Wood-Saxon wave functions instead of harmonic-oscillator wave functions, we obtain very similar matrix elements.

One limitation of our $0\nu\beta \beta$ sd-shell calculations is that the relativistic matrix element $\langle F^0_{211} \rangle$ is identically zero for harmonic-oscillator wave functions. This is not the case for Wood-Saxon wave functions, but the value obtained (of $\langle F^0_{211} \rangle = -0.004$) is too small to affect the results. Extending the model space to include multi-$\nu\beta\beta$ excitations is beyond the goals of the present publication and hence we follow a different approach to determine the $\langle F^0_{211} \rangle$ matrix element. Following Ref. [18] the conserved vector current (CVC) theory provides a relationship between $\langle F^0_{211} \rangle$ and $\langle F^0_{220} \rangle$:

$$\langle F^0_{211} \rangle = -\frac{1}{\sqrt{10}} \frac{E_y R}{\hbar c} \langle F^0_{220} \rangle,$$

where $E_y = 10.273$ MeV is the excitation energy of the isobaric analog state of the ground state of $^{20}\text{F}$ in $^{20}\text{Ne}$ [5]. The CVC relation Eq. (1) is expected to hold for the “exact” matrix elements but may break for matrix elements computed in a restricted model space using the impulse approximation as in our case. To quantify this effect we further relate the magnitude of the matrix element $\langle F^0_{220} \rangle$ to the experimentally determined E2 strength of the decay of the 10.273 MeV state to the ground state of $^{20}\text{Ne}$ assuming isospin symmetry:

$$\langle F^0_{220} \rangle = \frac{1}{R^2} \left( \frac{8\pi B(E2)}{e^2} \right)^{1/2}.$$

Adopting the experimental strength of $B(E2) = 0.306(84)$ e$^2$ fm$^4$, we obtain $\langle F^0_{220} \rangle = 0.184(25)$, while the sign of the matrix element is determined based on the shell-model results. The matrix elements $\langle F^0_{220}(1,1,1,1) \rangle$ and $\langle F^0_{220}(2,1,1,1) \rangle$ contain a slightly different radial dependence than the factor of $r^2$ appearing in $\langle F^0_{220} \rangle$. We assume that the ratios $\langle F^0_{220}(1,1,1,1) \rangle/\langle F^0_{220} \rangle$ and $\langle F^0_{220}(2,1,1,1) \rangle/\langle F^0_{220} \rangle$ are well described by the shell-model calculations. The full set of matrix elements obtained in this way are listed in the column labeled “SM+CVC+E2” in Table II.

The axial matrix elements $\langle F^0_{221} \rangle$, $\langle F^0_{221}(1,1,1,1) \rangle$, and $\langle F^0_{221}(2,1,1,1) \rangle$ could be affected by the quenching of the axial coupling constant observed in Gamow-Teller decays; see, e.g., Ref. [29]. However, previous studies have not shown conclusive evidence that such a quenching is also present in nonunique second-forbidden transitions [30–32]. Hence, we will consider in the following two different cases using either the bare value of $g_A = -1.27$ or the quenched value $g_A = -1.0$. The numbers in Table II have been obtained for $g_A = -1.27$.

In Fig. 13 we show the shape factor and $\beta$ spectrum of the second forbidden transition as a function of the electron kinetic energy. The theoretical log $ft$ values for the different cases are shown in the last column of Table III. Looking at the shape factor and $\beta$ spectrum one can notice important differences between the purely theoretical results, labeled “SM,” and those constrained by experimental information, labeled “SM+CVC+E2.” In the former case the shape is slightly distorted towards low energies compared with the allowed shape while in the latter case it is quite distorted towards high energies. This distortion originates from the important contribution of the relativistic matrix element $\langle F^0_{211} \rangle$ to the $w$, $w^2$, and $w^3$ terms that dominate the shape factor at high energies. For energies around the allowed $Q$ value, $w = 5.391/0.511 + 1 = 11.55$, the shape factor can...
be expressed as

\[
C(11.55) = 1.3 \times 10^{-6} \left\{ \left( F_{221}^{-} \right)^2 + 58.75 F_{221}^{-} F_{220}^{0} + 1231 \left( F_{221}^{0} \right)^2 + 1.405 F_{221}^{-} F_{220}^{0} + 81.081 F_{211}^{-} F_{220}^{0} + 1.777 \left( F_{220}^{0} \right)^2 \right\},
\]

where the ratios \( V_{KLL} \, (k_e, 1, 1)/V_{KLL} \) have been obtained from the shell-model calculations. One can see the important role of the relativistic matrix element \( V_{211}^{-} \) in determining the shape factor at high energies. For the two limiting cases considered above we have

\[
C(11.55) = 1.3 \times 10^{-6} \left\{ \left( F_{221}^{-} \right)^2 + 1.405 F_{221}^{-} F_{220}^{0} + 1.777 \left( F_{220}^{0} \right)^2, \quad V_{211}^{0} = 0, \right\},
\]

From the relative signs of the vector and axial matrix elements given in Table II, their interference is destructive for the first case and constructive in the second case. Furthermore, quenching leads to a small enhancement of the shape factor in the first case and a larger reduction in the second case.

As an additional validation of our theoretical approach, we have also computed the shape factor using a more general formalism that includes next-to-leading-order nuclear-matrix elements [33]. We find that these additional matrix elements have negligible influence on the shape factor of the forbidden transition.

In the next section, we combine the theoretical shape factors with the experimental \( \beta \) spectrum to determine the branching ratio of the forbidden transition.

**B. Likelihood fits to the experimental \( \beta \) spectrum**

To determine the branching ratio, we perform a likelihood fit to the experimental \( \beta \) spectrum between 5.0–8.0 MeV, in which we allow the normalization of the simulated spectra of the allowed and forbidden transitions to vary freely, while the background is modeled by a simple exponential function with two free parameters. While the normalization of the allowed transition is, in principle, fixed, in practice it is necessary to allow the normalization to vary because the GEANT4 simulation becomes inaccurate in the low-energy tail of the transmission window. We also allow for a small (\(<50 \) keV) constant energy shift to account for inaccuracies in the energy calibration.

We perform such a likelihood fit for each of the four forbidden shape factors shown in Fig. 13. For reference, we also perform fits assuming a forbidden shape factor of unity (i.e., allowed shape) and assuming no contribution from the forbidden transition. The results of these fits are summarized in Table IV. Apart from the fit that ignores the contribution of the forbidden transition, all fits have practically identical fit qualities, implying that the shape factor is essentially unconstrained by the experimental data. As a result, the branching ratios differ substantially, ranging from \( \sim 0.4 \times 10^{-5} \) to \( \sim 1.0 \times 10^{-5} \), with the smaller value being favored by the theoretical arguments given in Sec. IV B. We note that our result is consistent with the existing upper limit of \( \sim 10^{-5} \), which was obtained assuming an allowed shape [10].

When comparing the theoretical and experimental log \( ft \) values we find that the theoretical log \( ft \) values constrained by experimental data, labeled “SM+CVC+E2”, are consistent with experimental log \( ft \) values, while the purely theoretical log \( ft \) values, labeled “SM”, overestimate the half-life of the forbidden transition by a factor of \( \sim 1.6 \). We do not find major differences between the quenched and unquenched cases. In the following, we will adopt the shape factor given by the unquenched “SM+CVC+E2” model, shown in boldface in Table III, as this model is consistent with all the available experimental data including the CVC theory, the strength of

![Figure 13](image-url)

**FIG. 13.** Theoretical shape factor (a) and \( \beta \) spectrum (b) for the second forbidden transition as a function of the electron kinetic energy for the different cases discussed in text and Table II. The dashed vertical lines indicate the end-point energies of the allowed \( (Q_{\text{allowed}}) \) and forbidden \( (Q_{\text{forbidden}}) \) transition, respectively. The thin violet line shows the allowed spectral shape arbitrarily normalized to the “SM” case.
to the four spectra shown in Fig. 11 yields a branching ratio of

\[ \frac{\chi^2}{N} = 1.193 \]  

\[ p = 0.080 \]  

\[ b_{\beta} (\times 10^{-5}) = 0.41(8)(7) \]  

\[ \log ft = 10.89(11) \]  

\[ \log ft (theory) = 10.86 \]

Taking into account the uncertainties related to the normalization of the \( \beta \) spectrum discussed above, our result for the branching ratio of the forbidden transition is \( b_{\beta} = [0.41 \pm 0.08 (stat) \pm 0.07 (sys)] \times 10^{-5} \). This translates into \( \log ft = 10.89(11) \), where the statistical and systematic uncertainty have been added in quadrature.

V. ELECTRON CAPTURE RATE

The astrophysical importance of the forbidden transition was first pointed out in Ref. [2], where it was argued that the inverse \( 0^+ \rightarrow 2^+ \) transition could enhance the rate of electron capture on \(^{20}\text{Ne}\) in dense and hot astrophysical environments, thereby affecting the final evolution of stars that develop degenerate cores of oxygen and neon. Reference [2] also provided an estimate of the electron-capture rate based on the previous upper limit on the branching ratio of the forbidden transition [10] assuming an allowed shape. In the following, we generalize the calculation of the electron-capture rate to account for the forbidden shape.

At the high densities and temperatures of a degenerate oxygen-neon stellar core the nuclei are fully ionized and the
electrons form a relativistic and degenerate Fermi gas. The energy of the electrons follow the Fermi-Dirac distribution with chemical potential $\mu_e$ related to $\rho Y_e$, where $\rho$ denotes the matter density and $Y_e$ the electron fraction. The electron capture rate via the forbidden transition is given by

$$\lambda^{EC} = \frac{K}{2} \int_{q}^{\infty} C(w)p(w-q)^2F(Z,w)S_{\nu}(w,\mu_e)dw,$$

where $q$ is the positive $Q$ value of the transition in units of the electron mass, $S_{\nu}(w,\mu_e)$ is the Fermi-Dirac distribution, and $F(Z,w)$ is the Fermi function, where $Z$ is the charge number of the capturing nucleus. Screening corrections have been included in the calculation of the rate following Refs. [2,34].

We compute the shape factor of the electron-capture transition using the same expression as for $\beta$ decay, taking into account the different kinematics ($E_\nu = Q - E_\gamma$ for $\beta^-$ decay and $E_\nu = E_\gamma - Q$ for electron capture), using the same relative phases of the matrix elements as in Table II, and correcting for the trivial factor of $\sqrt{5}$ arising from the reversal of initial and final spins.

![FIG. 15. Normalized shape factors (a) and corresponding $\beta$-decay spectra (b) as a function of the electron kinetic energy. The two vertical lines correspond to the $Q$ values of the allowed and forbidden transitions, respectively. The experimental data constrains the spectra between these two values. For energies above the $Q$ value we show the shape factor for electron capture after dividing by a factor $\sqrt{5}$ in the upper panel. The lower panel shows spectra of captured electrons for the conditions of temperature, $\log_{10} T(K) = 8.6$ and density $\log_{10} \rho Y_e (g cm^{-3}) = 9.6$.](065805-10)

![FIG. 16. Electron capture rate on $^{20}Ne$ as a function of density at a temperature of $\log_{10} T(K) = 8.6$. We show the contributions of the forbidden transition studied in this work and two allowed transitions: One from the ground state of $^{20}Ne$ to the $1^+$ state in $^{20}F$ at 1.056 MeV, and one from the $2^+$ state of $^{20}Ne$ at 1.634 MeV to the ground state of $^{20}F$. The former dominates the rate at densities $\log_{10} \rho Y_e (g cm^{-3}) \gtrsim 9.67$, whereas the latter dominates at $\log_{10} \rho Y_e (g cm^{-3}) \lesssim 9.35$. However, at such densities the rate is so small that it has no influence on the evolution. For reference we also show the forbidden contribution as computed by Suzuki et al. [21].](065805-11)
with the two theoretical models that account for the energy dependence of the forbidden transition differ by less than 25%. Such a small difference has no significant impact on the evolution of degenerate oxygen-neon stellar cores. The rate computed assuming an allowed shape is smaller than the two other rates. The deviation becomes larger at higher densities, but remains within a factor of two in the relevant density range where the forbidden transition dominates over the allowed transition to the 1\textsuperscript{st} state. The rate determined in Ref. [21] based on a shell-model calculation, which uses the same USDB interaction as in the present work, is substantially smaller than the present rate. The deviation reaches a factor of \(~10\) at densities of log\(10\)\(\rho\)\(\text{g} \text{cm}^{-3}\) \(\sim\) 9.6. The origin of this discrepancy is not clear, but it is likely related to differences in the treatment of the forbidden transition, which lead to substantially different electron spectra (see their Fig. 5).

VI. DISCUSSION

Shell-model calculations are known to reproduce the strengths of second-forbidden, unique transitions in the sd shell within a factor of two or better [30,31]. A similar conclusion was also reached by Ref. [23] regarding the second-shell within a factor of two or better [30,31]. A similar rate computed assuming an allowed shape is smaller than the rate determined in Ref. [21] based on a shell-model calculation, which uses the known strength of the analog E2 transition in \(^{20}\text{Ne}\) together with the CVC theory reproduces the experimental half-life to within 10%. It would be of considerable interest to extend this comparison to the much weaker decay of \(^{36}\text{Cl}\) (log\(ft\) = 13.321(3) [35]), which is the only other known second-forbidden, nonunique transition in the sd shell. However, this is beyond the scope of the present study and is left for future work.

The \(2^+ \rightarrow 0^+\) transition in the \(\beta\) decay of \(^{20}\text{F}\) was only observed in a narrow energy range near the end point of the \(\beta\) spectrum. This, combined with limited statistics, a modest signal-to-background ratio, and a modest energy resolution, meant that the experimental data did not impose any useful constraints on the shape of the spectrum. Instead, the shape was determined from a combination of theoretical calculations and the known strengths of the analogous E2 transition in \(^{20}\text{Ne}\). This led to a branching ratio of \(b_\beta = [0.41 \pm 0.08\text{(stat)} \pm 0.07\text{(sys)}] \times 10^{-5}\) and a strength of log\(ft\) = 10.89(11). This makes the \(2^+ \rightarrow 0^+\) transition in the \(\beta\) decay of \(^{20}\text{F}\) the second-strongest, second-forbidden, nonunique transition ever measured, with the two other rates. The deviation becomes larger at higher densities, but remains within a factor of two.

Owing to its large strength, the forbidden transition between the ground state of \(^{20}\text{Ne}\) and \(^{20}\text{F}\) enhances the astrophysical electron-capture rate on \(^{20}\text{Ne}\) by several orders of magnitude at the elevated temperatures and densities achieved in contracting oxygen-neon stellar cores. This has significant implications on the final evolution of such stars as discussed elsewhere. Here, we have shown that the experimental data constrain the astrophysical capture rate to within better than 25%, which is fully sufficient to assess the astrophysical implications.

The experimental data did not impose any useful constraints on the shape of the forbidden \(\beta\) spectrum, which instead was determined based on shell-model calculations and the experimental \(B(E2)\) value of the analog transition in \(^{20}\text{Ne}\). Future experiments should aim to provide improved constraints on the shape of the forbidden \(\beta\) spectrum, although this will be very challenging.

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[17] P. Jals, T. Enqvist, V. Isoherranen, J. Joutsenvaara, J. Kutuniva, and P. Kuusiniemi, Callio lab, a new deep underground labora-


Correction: The inadvertent omission of a marker indicating “Featured in Physics” in the PDF format has been fixed.