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# Quantum entanglement in nuclear Cooper-pair tunneling with $\gamma$ rays

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While Josephson-like junctions, transiently established in heavy-ion collisions ($\tau_{\text{coll}} \approx 10^{-21}$ s) between superfluid nuclei—and through which Cooper-pair tunneling ($Q$-value $Q_{2n}$) proceeds mainly in terms of successive transfer of entangled nucleons—are deprived of the macroscopic aspects of a supercurrent, they display many of the special effects associated with spontaneous symmetry breaking in gauge space (BCS condensation), which can be studied in terms of individual quantum states and of tunneling of single Cooper pairs. From the results of studies of one- and two-neutron transfer reactions carried out at energies below the Coulomb barrier we estimate the value of the mean-square radius (correlation length) of the nuclear Cooper pair. A quantity related to the largest distance of closest approach for which the absolute two-nucleon tunneling cross section is of the order of the single-particle one. Furthermore, emission of $\gamma$ rays of (Josephson) frequency $\nu_J = Q_{2n}/h$ distributed over an energy range $\hbar/\tau_{\text{coll}}$ is predicted.

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*Introduction*. A pair of interacting electrons moving in time-reversal states $(\nu, \tilde{\nu})[\equiv (\mathbf{k}\uparrow, -\mathbf{k}\downarrow)]$ above a noninteracting Fermi sea whose only role is to block, through Pauli principle, states below the Fermi energy $\epsilon_F$ from participating in the two-particle system, lead to a bound state provided the interaction is attractive, no matter how weak it is [1].

At the basis of BCS superconductivity [2,3] one finds the condensation of strongly overlapping, very extended, weakly bound Cooper pairs corresponding to ordering in occupying momentum space, and not spacelike condensation of strongly bound clusters which undergo Bose condensation. In BCS condensation, the inner intrinsic structure of the pair, that is, the fact that it is made out of fermions entangled in time-reversal states, is the characterizing feature, with its energy gap for both single-pair translation and dissociation (see Refs. [4–7]), as it emerges from Schrieffer's trial wave-function $|\Psi_{\text{BCS}}\rangle = \prod_{\nu>0}(U'_\nu + e^{-2i\phi}V'_\nu P^\dagger_\nu)|0\rangle$ [8]. The associated spontaneously broken symmetry in the two-dimensional gauge space, is quantitatively measured by the generalized deformation (order) parameter $\alpha_0 = \langle\Psi_{\text{BCS}}|P^\dagger|\Psi_{\text{BCS}}\rangle = e^{-2i\phi}\alpha'_0$. The pair creation operator is defined as $P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\tilde{\nu}}$, where $a^\dagger_\nu(a^\dagger_{\tilde{\nu}})$ creates, acting on the vacuum state $|0\rangle$, a fermion (electron) moving in state $\nu(\tilde{\nu})$ while $\alpha'_0 = \sum_{\nu>0} U'_\nu V'_\nu$ measures the number of Cooper pairs, a quantity closely related to the pairing gap $\Delta' = G\alpha'_0(\approx 1$ meV), $G$ being the pairing coupling constant. The intrinsic, body-fixed frame of reference ($x'$ axis) subtends a gauge angle $2\phi$ with the laboratory axis $x$ (see, e.g., Fig. 11 Ref. [9]).

*Weakly coupled superconductors*. The Cooper-pair wave function can be written as $\langle \mathbf{r}_1\sigma_1, \mathbf{r}_2\sigma_2|\sum_{\nu>0} c'_\nu P^\dagger_\nu|0\rangle = \varphi_q(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{R}}\chi(\sigma_1,\sigma_2)$. The variable $\mathbf{r}\,(\mathbf{R})$ is the relative (center-of-mass) (c.m.) coordinate whereas $\mathbf{q}$ is the center-of-mass momentum, $\chi$ being the singlet spin function. For $q = 0$ [8,10]

$$\varphi_0(\mathbf{r}) \approx e^{-r/\xi}\cos k_F r, \tag{1}$$

the quantity

$$\xi = \frac{\hbar v_F}{\pi\Delta}, \tag{2}$$

being the correlation length ($\approx 10^4$ Å).

In the calculation of the Cooper-pair tunneling probability $p_2$ between two *weakly coupled superconductors (**S-S**), also known as the Josephson junction*, of typical width $d(\approx$10–30 Å $\ll \xi)$ one has to add the phased amplitudes before one takes the modulus squared. As a consequence, the probability $p_2$ of a pair going through the junction is comparable to the probability $p_1$ for a single electron ([11], see also Chap. 6 of Ref. [12]). This result is at the basis of the Josephson effect(s) [13–17]: (a) *unbiased junction*; the small but finite overlap of the condensed amplitudes $|\Psi_{\text{BCS}}(\ell)\rangle$ and $|\Psi_{\text{BCS}}(r)\rangle$ is sufficient to lock the associated gauge phase difference $[\phi_{\text{rel}}(\mathbf{R}) = \phi_\ell - \phi_r]$, function which acts as the velocity potential of a collective flow (center-of-mass momentum) superimposed on the Cooper pairs correlated intrinsic motion (1). The associated direct supercurrent of carriers of charge $q = 2e$ and maximum value $I_c = \frac{\pi}{e}\Delta_{\ell r}\frac{1}{R_b}$ is undamped because the internal degrees of freedom are frozen by the reduced pairing gap $\Delta_{\ell r} = \frac{\Delta_\ell\Delta_r}{\Delta_\ell+\Delta_r}$. In the above relation $\Delta_\ell$ and $\Delta_r$ are the pairing gap of the left and right superconductors with respect to the junction. Similarly concerning the gauge phases $\phi_\ell$ and $\phi_r$; (b) *biased junction*; when there is a dc voltage $V$, and, thus, an associated chemical potential difference $(\lambda_\ell - \lambda_r)$ across the junction, circulation of an alternating current of carriers $q = 2e$, critical value $I_c$ and of frequency $\nu_J = V2e/h$ is observed, while $\phi_{\text{rel}}$ precesses at the rate given by $\dot{\phi}_{\text{rel}} = (\lambda_\ell - \lambda_r)/\hbar = V2e/\hbar$. There is then an energy difference $\Delta E = V2e$ each time a Cooper pair tunnels from one side of the junction to the other, energy which must appear elsewhere. Being the process superconducting, it is free of dissipation. To leave the quasiparticle distribution unchanged, Cooper pairs can tunnel back and forth with the emission of a photon of frequency $\nu_J$. The Josephson junction not only converts a direct voltage into an alternating current, but also works as an oscillatory circuit. It radiates electromagnetic waves in the superhigh-frequency range.

The critical supercurrent $I_c$ (of typical value $\approx$2 mA) across an **S-S** junction is within a factor of $\pi/4$ equal to the **N-N** single electron carrier current for an applied equivalent potential bias $V_{\text{eq}} = (2\Delta/e)$ ($\approx$2 mV), S (N) indicating the superconducting (normal) phase of the metal. That is

$$I_c = \frac{\pi}{4}\frac{V_{\text{eq}}}{R_b}, \tag{3}$$

where $R_b(\approx 1\ \Omega)$ is the resistance of the junction ($I_c \approx 1.6$ mA). A relation which testifies to the correctness of $p_2 \approx p_1$, and constitutes one of the pillars on which the validity of the BCS description of superconductivity rests. Another one is provided by the photons of frequency,

$$\nu_J = K_J V, \tag{4}$$

emitted by a biased S-S junction. The Josephson constant, inverse of the flux quantum (fluxon) is $K_J = 2e/h$. For voltage differences across the junction of $\approx$1 mV one has $\nu_J \approx 0.5$ THz.

It is of note that in the tunneling process between two superconductors in which a bias of value $V \gtrsim 2\Delta/e$ is applied to the junction, a momentum $\gtrsim q = \hbar/\xi$ is given to the center of mass of $\varphi_q(\mathbf{r})$ and, as a result, Cooper pairs are broken and quasiparticle excitations created–thus the labeling (**S-Q**) given in the literature to such processes–through which a normal (dissipative) current of carriers $q = e$ flows [18]. In other words, for $T = 0$ one is in the presence of processes connecting a ground state (S) with ground and excited states (Q). The importance of this fact in connection with the Josephson-like junction transiently formed in heavy-ion reactions between superfluid nuclei, becomes apparent below.

*Cooper-pair tunneling in nuclei*. Recently, a breakthrough on the subject was made through the study of one- and two-neutron transfer reactions with heavy-ion collisions in inverse [19] and direct [20] kinematics, enabled by the use of magnetic and $\gamma$-ray spectrometers,

$$^{116}\text{Sn} + {}^{60}\text{Ni} \rightarrow \begin{cases} ^{115}\text{Sn} + {}^{61}\text{Ni} \quad (Q_{1n} \approx -1.74\ \text{MeV}), & \text{(5a)} \\ ^{114}\text{Sn} + {}^{62}\text{Ni} \quad (Q_{2n} \approx 1.307\ \text{MeV}). & \text{(5b)} \end{cases}$$

These reactions were carried out for twelve bombarding energies in the range of 140.60 MeV $\leqslant E_{\text{c.m.}} \leqslant$ 167.95 MeV. That is, from energies above the Coulomb barrier ($E_B =$ 157.60 MeV), to well below it. While the Cooper-pair transfer channel [(5b)], is dominated by the ground-ground state transition, the single-particle transfer one is inclusive. In fact, the theoretical calculations of the differential cross section associated with channel [(5a)] indicate the incoherent contribution of a number of quasiparticle states of $^{61}$Ni lying at energies $\lesssim$2.640 MeV ([19,21]). A value which is consistent with twice the value of the pairing gap of Ni. In other words, in the case of the reaction (5a), we are in the presence of a S-Q-like transfer. Making use of the relation (2), as well as of the values $(v_F/c) \approx 0.3$ and $\Delta \approx 1.3$–$1.5$ MeV, one obtains $\xi \approx 13.6$ fm (within this context see for example Fig. 7, Appendix A of Ref. [22]).

The analysis of the data associated with the reactions [(5a)] and [(5b)] carried out in Refs. [19,20] makes use of a powerful semiclassical approximation in which the optical potential employed was microscopically calculated in terms of the interaction energy per unit area, proximity potential proportional to the surface tension and the reduced radius, regarding the real part ([23] Eqs. (30) and (40)–(43), pp. 111 and 114). The imaginary part was worked out in terms of first-order transition probabilities making use of the same microscopic form factors used in the analysis of the data [24–26]. The resulting potentials have been extensively tested throughout the mass table [27–30]. The short wavelength of relative motion (de Broglie reduced wavelength $\lambda\!\!\!^{-} = 0.36/2\pi$ fm $\approx 0.06$ fm), allows to accurately determine the distance of closest approach $D_0$ for each bombarding energy, by calculating the corresponding classical trajectory as a solution of the equations of motion associated with the real part of the optical potential plus the Coulomb potential. The accuracy of the resulting connection between $E_{\text{c.m.}}$ and $D_0$ was demonstrated by the comparison between the theoretical and experimental values of $\sigma_{\text{el}}/\sigma_{\text{Ruth}}$ displayed in the upper part of Fig. 3 of Ref. [19]. Making use of the $U, V$ occupation amplitudes for both Sn and Ni, as well as the optical potential given in Refs. [19,21] we have calculated, within the framework of first- and second-order distorted-wave Born approximation (DWBA) [31], the absolute one- and two-nucleon transfer differential cross sections. In the second case, including both successive (dominant channel) and simultaneous transfer, properly corrected by nonorthogonality. Theory is compared with the experiment in Table I. As expected [19], the results provide an overall account of the experimental findings.

From direct inspection of this table it emerges that the distance of closest approach lying within the interval 13.12 fm $\leqslant D_0 \leqslant$ 13.49 fm is the largest one for which $d\sigma/d\Omega|_{2n}$ is, within a factor of about $[0.6 \approx (\pi/4)^2]$ of the same order of $d\sigma/d\Omega|_{1n}$. In keeping with (1) and (2) one can posit that the above interval provides a sensible bound to the size of the nuclear Cooper-pair correlation length. Already increasing $D_0$ by $\approx$0.6 fm ($D_0 = 14.05$) $\sigma_{1n}$ becomes a factor of 6 larger than $\sigma_{2n}$. A signal indicating that stretching the transferred Cooper pair to larger dimensions ruptures it, quenches its pairing gap and unfreezes the quasiparticle degrees of freedom. Said differently, a consequence of forcing Cooper-pair partners, in the dominant successive transfer process, to be at a relative distance longer than $\xi$. This leads to a strain which plays a role similar to that played by applying a momentum $q \approx 1/\xi$ (associated with the critical bias $V_{\text{eq}} = 2\Delta/e$) to the center of mass of the Cooper pairs, resulting in the transition

TABLE I. Center-of-mass absolute differential cross section at 140° [19,20,46] associated with the reactions (5). In brackets the results of the theoretical calculations carried out as explained in the text. For the 12 bombarding energies ($E = E_{\mathrm{c.m.}}$) also the distance of closest approach $D_0$ is indicated.

| | $E = 140.6$ MeV | $E = 145.02$ MeV | $E = 146.10$ MeV | $E = 148.10$ MeV | $E = 150.62$ MeV | $E = 151.86$ MeV |
|---|---|---|---|---|---|---|
| $D_0$ (fm) | 14.8 | 14.39 | 14.24 | 14.05 | 13.81 | 13.70 |
| $\sigma_{1n}^{\mathrm{exp}}(\sigma_{1n}^{\mathrm{th}})$ (mb/sr) | 1.24 (1.10) | 2.13 (2.01) | 2.32 (2.29) | 3.00 (2.96) | 3.50 (3.75) | 5.03 (4.51) |
| $\sigma_{2n}^{\mathrm{exp}}$ $(\sigma_{2n}^{\mathrm{th}})$ (mb/sr) | 0.07(0.05) | 0.23 (0.19) | 0.31 (0.26) | 0.5 (0.44) | 1.00 (0.87) | 1.83 (1.22) |
| | $E = 154.26$ MeV | $E = 158.63$ MeV | $E = 162.11$ MeV | $E = 164.4$ MeV | $E = 164.8$ MeV | $E = 167.95$ MeV |
| $D_0$ (fm) | 13.49 | 13.12 | 12.83 | 12.66 | 12.62 | 12.39 |
| $\sigma_{1n}^{\mathrm{exp}}(\sigma_{1n}^{\mathrm{th}})$ (mb/sr) | 7.25 (6.03) | 9.70 (9.08) | 7.88 (9.51) | 5.92 (4.64) | 4.83 (4.53) | <0.7 (0.25) |
| $\sigma_{2n}^{\mathrm{exp}}$ $(\sigma_{2n}^{\mathrm{th}})$ (mb/sr) | 2.58 (2.35) | 6.80 (7.54) | 6.11 (8.85) | 4.08 (2.34) | 3.48 (1.68) | <0.25 (0.07) |

from the S-S transfer regime to the S-Q one. As a result, we choose $D_0 = 13.49$ fm as a representative value for $\xi$ of the transferred Cooper pair.

*Nuclear analog of radiating Josephson junction*. As stated before, when the two superconducting elements of a junction are at a different electric potential, the transfer of a pair of electrons from one side (e.g., $\ell$) to the other one ($r$) involves an energy change of $(2e)V$. If the process is truly a superfluid process, free of dissipation, this energy must appear elsewhere as a unit. In fact, it appears as a photon of energy $h\nu = 2e\times V$ (radiofrequency) in keeping with (4), and as experimentally observed (see, e.g., Ref. [32] and references therein).

In the nuclear case and in connection with the reaction [(5b)] for bombarding conditions for which $D_0 = 13.49$ fm (namely $E_{\mathrm{c.m.}} \approx 154.26$ MeV, and $\tau_{\mathrm{coll}} \approx \xi/(2E_{\mathrm{c.m.}}/\mu_i)^{1/2} \approx 0.5\times10^{-21}$ s) transfer takes place few MeV below the Coulomb barrier. Consequently, the absorptive component of the optical potential plays essentially no role in the process, and tunneling takes place lossless, free of dissipation. Being the bombarding energy $\approx$3.9 MeV/$A$ ($E_{\mathrm{lab}} = 452.5$ MeV), that is an order of magnitude smaller than the Fermi energy, one can expect that there can be time for the two neutrons to be transferred back and forth about three times. That is, for about two ($\approx$1.5) cycles of the quasielastic process,

$$^{116}\mathrm{Sn} + {}^{60}\mathrm{Ni} \to {}^{114}\mathrm{Sn} + {}^{62}\mathrm{Ni} \to {}^{116}\mathrm{Sn} + {}^{60}\mathrm{Ni}. \tag{6}$$

Due to the fact that nuclear Cooper pairs carry an effective charge $(e)_{\mathrm{eff}} \approx (-2eZ/A)$, one expects the transient Josephson-like nuclear junction to emit $\gamma$ rays of frequency $\nu = Q_{2n}/h$ (=1.307 MeV/$h$). Because of the short collision time ($\tau_{\mathrm{coll}}$) the radiated photons will display a width ($\approx\hbar/\tau_{\mathrm{coll}}$). Due to the recoil of the $\ell$(Sn)-$r$(Ni) nuclear superconducting junction associated with Cooper-pair tunneling, the corresponding line shape will be distorted with respect to a Gaussian-like shape.

In what follows we calculate the $\gamma$-emission cross section in terms of a macroscopic formulation of the (ac) Josephson effect, particularly, suited to be used in connection with the nuclear case.

Concerning the search for nuclear analogs of the Josephson effect see (Refs. [33–39], see also Ref. [23]).

*Macroscopic calculation of dipole emission*. Making use of $\alpha_0 = e^{-2i\phi}\alpha_0'$ one can introduce the density of superconducting electron (fermion) pairs,

$$\Psi^*\Psi = \frac{\alpha_0'}{\mathcal{V}} = n_s', \tag{7}$$

in terms of the pair probability amplitude [40,41],

$$\Psi = e^{-i\phi}\sqrt{n_s'}, \tag{8}$$

where $\mathcal{V}$ is an appropriate volume element. Both $n_s'$ and $\phi$ can be functions of space (and time), and their variation determines the motion of the BCS condensate, e.g., the supercurrent. Since the pairs are in the same state and must, therefore, behave in an identical fashion, the equations of motion of the macrostate must coincide with the equation of motion for any single pair of this state [42]. In other words, due to its unique coherence properties the condensed (superfluid) portion of the superconductor behaves, like a single quantum particle of mass and charge twice that of an electron.

It is then sensible to expect that the dynamical behavior of a Josephson junction—right ($r$) and left ($\ell$) weakly coupled superconductors—would be similar to that of two quantum levels weakly coupled to one another via an external field [43]. Considering the situation in which the tunneling interaction is relatively constant over a coherence length [44], the electrodynamics of a radiating Josephson junction is analogous to that of a two-level atom placed in a static external field, role which in the present case is played by the tunneling interaction inducing nonresonant transitions between the two quantum levels. These transitions give rise to an induced dipole moment whose oscillations generate the coherent Josephson radiation field, the intensity of the emitted radiation being proportional to the number of Cooper pairs that are involved in the tunneling process quantity squared, the frequency being that defined in Eq. (4).

A similar, incipient superradiant Josephson-like phenomenon is expected to arise in the case of the nuclear heavy-ion reaction under discussion from an ensemble of correlated Cooper pairs [$\alpha_0' \approx 8$ (2), $^{116}$Sn ($^{60}$Ni)] undergoing the coherent back and forth quasielastic Cooper-pair transfer process. In what follows the associated $\gamma$-emission probability is calculated.

According to Fermi's golden rule, the rate of spontaneous emission between two levels in the dipole approximation can be written as ([45], p. 340)

$$\frac{dP_{\text{if}}}{dt}=\frac{4\omega_{\text{if}}^3|\langle i|\mathbf{d}|f\rangle|^2}{3\hbar c^3}, \tag{9}$$

where $\omega_{\text{if}}=2\pi/\mathcal{T}$ is the emission frequency, $\mathcal{T}$ being the associated period, $\mathbf{d}=q\mathbf{r}$ the dipole moment operator and $q$ the charge.

In connection with the reaction [(5b)] $i\equiv B(=A+2)+b\to f\equiv A+a(=b+2)$, and $q=2e_{\text{eff}}=-2\times e(Z_b+Z_B)/(A_b+A_B)$, where $(A_b,Z_b)\equiv(60,28)$ and $(A_B,Z_B)\equiv(116,50)$, one obtains $q=-2\times(78/176)e=-e\times0.89$, and $d=-e\times0.89\times13.49$ fm $=-e\times12.01$ fm. Let us now calculate $\frac{dP_{\text{if}}}{dt}=\mathcal{N}/\mathcal{T}$, where $\mathcal{N}$ is the number of photons emitted per cycle,

$$\mathcal{N}=\mathcal{T}\times\frac{dP_{\text{if}}}{dt}=\frac{8\pi}{3}\frac{(\hbar\omega_{\text{if}})^2d^2}{(\hbar c)^3}\approx3.71\times10^{-4}, \tag{10}$$

and $\hbar\omega_{\text{if}}=Q_{2n}=1.307$ MeV. Making use of the experimental value (see Table I), $d\sigma_{2n}(E_{\text{c.m.}}=154.26\text{ MeV})/d\Omega|_{\theta_{\text{c.m.}}=140^\circ}\approx2.58$ mb/sr, one expects the associated $\gamma$ radiation to be emitted perpendicular to the reaction plane with a cross-section $d\sigma/(d\Omega dE_\gamma)=\mathcal{N}d\sigma_{2n}/d\Omega|_{\theta_{\text{c.m.}}=140^\circ}\delta(E_\gamma-Q_{2n})\approx0.96$ $\mu$b/sr $\delta(E_\gamma-Q_{2n})$. In other words, one expects a (reduced) strength function of centroid 1.307 MeV, width $\hbar/\tau_{\text{coll}}\approx1.3$ MeV and energy integrated area of 0.96 $\mu$b/sr.

*Dipole radiation: microscopic calculation.* A similar calculation of the $\gamma$-emission quasielastic process, this time fully microscopic, was carried out extending the second order DWBA formalism employed in the calculation of the two-nucleon transfer absolute differential cross sections displayed in Table I to include the coupling to the electromagnetic field in the dipole approximation.

The $T$ matrix associated with the successive transfer of the Cooper pairs, that is, half of a cycle of the process leading to the result (10) and which contributes essentially all of the corresponding cross section, can be written in the post-post representation as

$$T_{m_\gamma}(\mathbf{k}_f,\mathbf{k}_i)=2\sum_{\nu,\nu'}B_\nu^{(A)}B_{\nu'}^{(b)}\int\chi_f^*(\mathbf{r}_{Bb},\mathbf{k}_f)\big[\phi_{j_f}(\mathbf{r}_{A_1})\phi_{j_f}(\mathbf{r}_{A_2})\big]_0^{0*}U^{(A)}(r_{b1})\big[\phi_{j_f}(\mathbf{r}_{A_2})\phi_{j_i}(\mathbf{r}_{b_1})\big]_M^K d^1_{m_\gamma}(\mathbf{r}_{O1})d\mathbf{r}_{Cc}\,d\mathbf{r}_{b_1}\,d\mathbf{r}_{A_2}$$
$$\times\int G(\mathbf{r}_{Cc},\mathbf{r}'_{Cc})\big[\phi_{j_f}(\mathbf{r}'_{A_2})\phi_{j_i}(\mathbf{r}'_{b_1})\big]_M^{K*}U^{(A)}(r'_{c2})\big[\phi_{j_i}(\mathbf{r}'_{b_2})\phi_{j_i}(\mathbf{r}'_{b_1})\big]_0^0\chi_i(\mathbf{r}'_{Aa},\mathbf{k}_i)d\mathbf{r}'_{cC}\,d\mathbf{r}'_{b_1}d\mathbf{r}'_{A_2}, \tag{11}$$

where $B_j^{(i)}=(\sqrt{j+\frac{1}{2}}U_j^{(i)}V_j^{(i)})$ is the two-nucleon transfer spectroscopic amplitude (see e.g., Ref. [31]; see also Ref. [13]), while $U^{(A)}(r)$ is the mean-field potential[1] mediating the successive transfer process $B(=A+2)+b\to F(=A+1)+f(=b+1)\to A+a(=b+2)$. The Green's function $G(\mathbf{r}_{Cc},\mathbf{r}'_{cC})$ propagates the intermediate channel $(F,f)$ (no asymptotic waves), while $\chi_i$, $\chi_f$ are the distorted waves describing the relative motion of the heavy ions in the initial $(B,b)$ and final $(A,a)$ channels, the momenta $k_i$ and $k_f$ ensuring energy conservation. The dipole operator is defined as

$$d^1_{m_\gamma}=q\sqrt{\frac{4\pi}{3}}\mathcal{Y}_{1m_\gamma}(\mathbf{r}_{O1}), \tag{12}$$

where $\mathcal{Y}_{1m_\gamma}(\mathbf{r}_{O1})$ is the vector spherical harmonic of order one, $\mathcal{Y}_{1m_\gamma}(\mathbf{r}_{O1})=r_{O1}Y^1_{m_\gamma}(\hat{r}_{O1})$, and $r_{O1}$ is the coordinate of one of the transferred neutrons measured from the center of mass.

The $\gamma$-strength function (double differential cross section) associated with (11) can be written as

$$\frac{d^2\sigma}{d\Omega\,dE_\gamma}=\left(\frac{\mu_i\mu_f}{(2\pi\hbar^2)^2}\frac{k_f}{k_i}\right)\left(\frac{8\pi}{3}\frac{E_\gamma^2}{(\hbar c)^3}\right)|T_{m_\gamma}(\mathbf{k}_f,\mathbf{k}_i)|^2$$
$$\times\delta[E_\gamma+E_f-(E_i+Q)], \tag{13}$$

where $E_\gamma=\hbar\omega_{if}$, $k_i=(2\mu_iE_i)^{1/2}/\hbar$ and $k_f=(2\mu_fE_f)^{1/2}/\hbar$, $E_i$ and $E_f$ being the (c.m.) kinetic energy in initial and final channels.

In addition to the analytic prefactors describing the electromagnetic and kinematical phase spaces, the strength function (13) depends on the photon energy through the distorted waves and the effective form factors which, in channel $[F(A+1)]$, $[f(=b+1)]$, restrict the integrations to the region of overlap between the partner nucleons of the tunneling Cooper pair. In other words, for the overlap region associated with the largest relative distance between the two ions in which the normal and abnormal densities are simultaneously present. That is, the distance of closest approach corresponding to the correlation length $\xi$.

Making the ansatz $\theta_{\text{c.m.}}=0^\circ(\hat{k}_i=\hat{k}_f=\hat{z})$, $m_A\approx m_B$, $m_b\approx m_a\gg1$, and substituting the distorted waves by plane waves one obtains for small momentum transfer ($q\to0$), $T\approx\exp[-(Q-E_\gamma)^2/\Delta E^2]$, and the FWHM of the line shape is $\Delta E\approx\sqrt{3}(\hbar/\tau_{\text{coll}})$ ($\approx2.30$ MeV). The fact that, in a Josephson junction, the two superconductors $S_\ell$ and $S_r$ are macroscopic objects at rest implies that the $\delta$ function in (13) is replaced by $\delta(\omega-(2e)V/h)$ which in the nuclear case translates into $\delta(E_\gamma-Q_{2n})$.

The $\gamma$-strength function (13) was worked out making use of microscopic form factors [see Eq. (11)]. They were obtained from the coherent summation of products of single-particle wave functions weighted by the two-nucleon spectroscopic amplitudes. These wave functions were calculated with the help of the mean-field potentials $U^{(i)}$, potentials which also act in the transfer process, propagated from the initial to the final channel by the Green's function. The distorted

[1]It is of note that the mean-field potentials $U^{(A)}$ and $U^{(b)}$ are those used in the calculation of the single-particle wave functions appearing in Eq. (11).

waves $\chi$ were determined with the help of the microscopic optical potential of Ref. [19]. Up to 150 partial waves were included in the calculation. The final results are shown in Fig. 1(a) in terms of a dashed line. It describes a $\gamma$-strength function with centroid, FWHM and energy integrated area of 4 MeV, 5 MeV, and 5.18 $\mu$b/sr, respectively, associated with a dipole moment $\langle d\rangle = -e\times 9.36$ fm ($\langle r\rangle \approx 10.52$ fm).

Multiplying these results by $\left(\frac{8\pi}{3}\frac{(1.307)^2\,\mathrm{MeV}^2}{(\hbar c)^3}\right)\left(\frac{8\pi}{3}\frac{E_\gamma^2}{(\hbar c)^3}\right)^{-1}$ one obtains a Gaussian-like reduced $\gamma$-strength function [Figs. 1(a) and 1(b) continuous line]. The associated centroid, FWHM, and energy integrated area being: 1.1 MeV, 3.6 MeV, and 0.57 $\mu$b/sr, respectively. Quantities which can be compared at profit with the corresponding results of the macroscopic calculations.[2] Summing up, both the centroid, width as well as the line shape of the $\gamma$-strength function are distorted as compared to the simple dipole macroscopic estimate let alone in relation to that observed in the radiofrequency emission from a Josephson junction (see e.g., Ref. [32]). All this without jeopardizing the validity of the nuclear analogy.

From the comparison of the estimate of the correlation length of 13.49 fm made by following $\sigma_{2n}/\sigma_{1n}$ as a function of the bombarding energy $E_{\mathrm{c.m.}}$ ($D_0$) and determining $\sigma_{2n}/\sigma_{1n}|_{(D_0)_{\max}} \gtrsim 0.6$, and that obtained from the quantum-mechanical calculation of the value of the dipole operator (12), i.e., of the distance of 10.52 fm over which the partner nucleons of the transferred Cooper pair are correlated in the associated successive tunneling process, one can ascribe an error to the theoretical estimate of the correlation length leading to $\xi \approx 12.0 \pm 1.5$ fm.

*Conclusions.* The special effect found in superconductivity by which a dc voltage $V$ applied across a junction between two superconductors does not determine the intensity of the supercurrent (Ohm's law) circulating through it, but

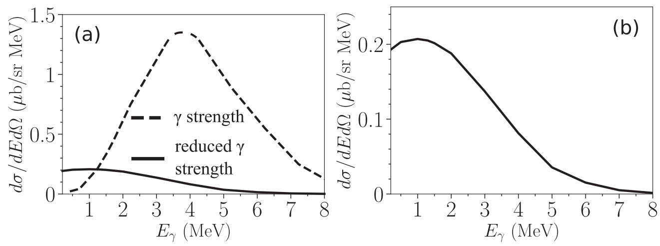


FIG. 1. (a) Double differential cross section for $\gamma$ emission at $\theta_{\mathrm{c.m.}} = 140^\circ$ as a function of the energy of the emitted $\gamma$ ray, calculated with Eqs. (11) and (13) (dashed curve). The reduced strength (continuous curve) has been obtained by dividing out from $d\sigma/d\Omega\, dE_\gamma$ the phase-space factor $\sim E_\gamma^2$, and multiplying it by the corresponding quantity with $E_\gamma = 1.307$ (MeV) (see the text for details). The reduced $\gamma$-strength function is shown in (b) with a different scale where the width and the position of the centroid are more apparent.

the frequency of an alternating supercurrent ($\nu_J = (2e)V/h$), finds its nuclear analog in the electromagnetic radiation predicted to be emitted in a quasielastic heavy-ion collision between two superfluid nuclei in terms of $\gamma$ rays of frequency $\nu = Q_{2n}/h$. For the particular reaction studied, and selecting the bombarding energy for which the distance of closest approach is approximately equal to the correlation length $\xi \approx 13.5$ fm (largest of the measured distances of closest approach for which $\sigma_{2n} \approx \sigma_{1n}$ within a factor of two), theory predicts for the (energy integrated) reduced $\gamma$-strength function $d\sigma_\gamma/d\Omega|_{\theta_{\mathrm{c.m.}}=140^\circ} \approx 0.57$ $\mu$b/sr ($\nu_J \approx 1.1$ MeV/$h$) corresponding to an observable (energy integrated) $\gamma$-strength function $d\sigma_\gamma/d\Omega|_{\theta_{\mathrm{c.m.}}=140^\circ} \approx 5.18$ $\mu$b/sr, peaked at $\approx$4 MeV. It can be concluded that a nuclear analog to the (ac) Josephson effect has been identified.

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[2]Making use of the value $\langle d\rangle = -e\times 9.36$ fm resulting from (11), one can estimate, from the macroscopic prediction, a microscopic one. Namely, $(9.36/12.01)^2\times 0.96$ $\mu$b/sr $\approx 0.58$ $\mu$b/sr. A result which testifies to the validity of the separability between the $\gamma$ process (number of photons $\mathcal{N}$) and the two-nucleon transfer one ($\sigma_{2n}$), assumed in the macroscopic model.

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*Correction:* The first sentence of the abstract contained a syntax error and has been fixed to achieve clarity.