Feedback Cooling of the Normal Modes of a Massive Electromechanical System to Submillikelvin Temperature

A. Vinante,1,2, M. Bignotto,3,4 M. Bonaldi,1,2 M. Cerdonio,3,4 L. Conti,3,4 P. Falferi,1,2 N. Liguori,3,4 S. Longo,5 R. Mezzana,6,2 A. Ortolan,5 G. A. Prodi,6,2 F. Salemi,6,2 L. Taffarello,4 G. Vedovato,4 S. Vitale,6,2 and J.-P. Zendri4

1Istituto di Fotonica e Nanotecnologie, CNR-Fondazione Bruno Kessler, 38100 Povo, Trento, Italy
2INFN, Gruppo Collegato di Trento, Sezione di Padova, 38100 Povo, Trento, Italy
3Dipartimento di Fisica, Università di Padova, 35131 Padova, Italy
4INFN, Sezione di Padova, 35131 Padova, Italy
5INFN, Laboratori Nazionali di Legnaro, 35020 Legnaro, Padova, Italy
6Dipartimento di Fisica, Università di Trento, 38100 Povo, Trento, Italy

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The prospect of observing quantum behavior and investigating decoherence in macroscopic mechanical resonators [1,2] requires cooling to ultralow temperatures, such that the thermal energy becomes comparable to the quantum energy. In addition to conventional refrigeration, this goal requires the use of feedback cooling techniques [3], unless the frequency is as large as several hundred MHz. Various authors have recently reported advances in cooling mechanical resonators, using either optomechanical [4–10] or electromechanical [11,12] techniques, and implementing either active feedback [4–7] or dynamical back-action effects [8–12]. These experiments involved mostly nanomechanical or micromechanical resonators, and have demonstrated cooling capability down to mK temperatures [6]. Cooling of a gram-scale optical spring resonator to a few mK has been demonstrated as well using the techniques developed for interferometric gravitational wave (GW) detectors [7]. In this Letter, we show that very efficient cooling can be achieved even in much larger systems, exploiting the techniques developed for resonant-mass GW detectors, based on electromechanical transducers coupled to superconducting quantum interference device (SQUID) sensors. In this case, one can take advantage of the larger quality factor achievable in macroscopic systems with respect to micromechanical ones. As experimental demonstration, we simultaneously cool the three electromechanical normal modes of the ton-scale resonant-bar GW detector AURIGA. Our cooling technique is based on an electronic feedback directly applied to the input circuit of the SQUID.

AURIGA represents the state-of-art in the class of resonant GW detectors [13], and has been in continuous operation from year 2004, searching for galactic astrophysical events in collaboration with a world network of detectors [14]. It is located in Padua (Italy) and is based on a 2.2 × 10^3 kg, 3 m long bar made of a low-loss aluminum alloy (Al5056), cooled to liquid helium temperatures. The fundamental longitudinal mode of the bar, sensitive to gravitational waves, has an effective mass M = 1.1 × 10^3 kg and a resonance frequency \( \omega_B/2\pi = 900 \) Hz. According to the equipartition theorem, the rms amplitude of the resonator motion is given by \( x_{\text{rms}} = \langle x^2 \rangle^{1/2} = (k_B T/M \omega_B^2)^{1/2} \), where \( k_B \) is the Boltzmann constant and \( T \) is the temperature. For the AURIGA bar, the rms thermal motion is \( x_{\text{rms}} = 4 \times 10^{-17} \) m at \( T = 4.2 \) K. The bar resonator motion is detected by a displacement sensor with a sensitivity of order several 10^{-20} m/\sqrt{Hz} over a ~100 Hz bandwidth. This sensitivity is accomplished by a multimode resonant capacitive transducer [15] combined with a very low noise dc SQUID amplifier [16] (Fig. 1). In this scheme, the bar resonator is coupled to the fundamental flexural mode of a mushroom-shaped lighter resonator, with 6 kg effective mass and the same resonance frequency. As the mechanical energy is transferred from the bar to the lighter resonator, the motion is magnified by a factor of roughly 15. A capacitive transducer, biased with a static electric field of 10^7 V/m, converts the differential motion between bar and mushroom resonator into an electrical current, which is finally detected by a low noise dc SQUID amplifier through a low-loss high-ratio superconducting transformer. The transducer efficiency is further increased by placing the resonance frequency of the electrical LC circuit close to the mechanical resonance frequencies [15], at 930 Hz.

The detector can then be simply modeled as a system of three coupled resonators: its dynamics is described by
three normal modes at separate frequencies, each one being a superposition of the bar and transducer mechanical resonators and the LC electrical resonator [13]. As seen from the SQUID sensor, each mode $k (k = 1, 2, 3)$ is modeled as a RLC series electrical mode with an effective inductance $L_k$, capacitance $C_k$, and resistance $R_k$ (Fig. 2). The total inductance $L_k$ includes also the input inductance $L_{in}$ of the SQUID amplifier. Around the resonance frequency of each mode $\omega_k/2\pi$, with $\omega_k = (L_k C_k)^{-1/2}$, the complex impedance of the circuit is expressed by $Z_k(\omega) = R_k + i\omega L_k + 1/(i\omega C_k)$. We point out that, although the modes appear as purely electrical as seen from the SQUID, their dynamics actually includes the full motion of both mechanical resonators. In fact, the current in the electrical circuit is linearly related with the mechanical motion of bar and transducer, and the effective impedance parameters $L_k$, $C_k$, $R_k$ of each normal mode are determined by the mechanical and electrical parameters of all resonators [13]. Thus, cooling the normal modes of the system implies cooling the motion of the mechanical resonators. The effective impedance parameters $L_k$, $C_k$, $R_k$ are experimentally estimated by measuring the current $I_s = V_{cal}/Z_k$ in response to a calibration voltage signal $V_{cal}$ injected through a small calibration coil in series to the SQUID input coil (see Fig. 2). The inductances $L_k$ of the three modes are respectively $1.66 \times 10^{-4}$ H, $1.23 \times 10^{-5}$ H and $8.12 \times 10^{-6}$ H. The mode resonance frequencies $\omega_k/2\pi$ are 865 Hz, 914 Hz, 953 Hz, and the quality factors $Q_k \equiv \omega_k L_k/R_k$ are $1.2 \times 10^6$, $0.88 \times 10^6$, and $0.77 \times 10^6$.

The effective temperature $T_k$ of each mode is proportional to the mean square current $\langle I_k^2 \rangle$ induced by thermal fluctuations driving the mode, according to the equipartition theorem:

$$\langle I_k^2 \rangle = \frac{k_B T_k}{L_k}. \quad (1)$$

To reduce the mode temperature, we implement an electronic feedback cooling (Fig. 2), or cold damping, technique [17]. The SQUID output voltage $V_o = A I_s$ is passed through a passive low-pass filter $D(\omega)$ with cutoff frequency at 200 Hz, and the current $I_D = AD I_s$ is fed back to the signal circuit. The low-pass filter is used to phase-shift by $\pi/2$ the feedback current $I_D$, which is then proportional to the derivative of the oscillating current of the mode. As a consequence, the effect of the feedback current is equivalent to that of a viscous damping, similarly to the case of a mechanical oscillator subjected to a force proportional to its velocity. This additional damping can be represented by an equivalent resistor $R_D$ in series with $R_k$, which can be calculated from the model of Fig. 2:

$$R_D = \text{Re}\left(\frac{iAD}{1 - AD/\omega L_{in}}\right). \quad (2)$$

In particular, in our experiment $R_D \equiv |AD|\omega L_{in}$, as $|AD| \ll 1$ and $AD$ is almost purely imaginary. Inductance and capacitance of the mode are not significantly modified, provided that the feedback current phase-shift is close to $\pi/2$. Therefore, we write the total impedance of the circuit under feedback cooling conditions, for $\omega \approx \omega_k$, as $Z'_k(\omega) = R_k + R_D + i\omega L_k + 1/(i\omega C_k)$. As customary, the relative strength of the feedback damping can be expressed by the ratio of the feedback to the intrinsic damping resistance (referred to the $k$th mode) $g_k = R_D/R_k$. Then, the quality factor of the $k$th mode under feedback cooling is reduced to $Q'_k = Q_k/(1 + g_k)$.

According to the fluctuation-dissipation theorem, the thermal noise in the $k$th normal mode is generated by a voltage noise source $V_{th}$ with single-sided power spectral density $S_{V_{th}} = 4k_B T_0 R_k$, where $T_0$ is the temperature of the thermal bath. This voltage noise is the effect of the interaction of the resonators with the microscopic degrees of freedom of the thermal bath, and therefore it is not affected by the feedback cooling. For $\omega \approx \omega_k$, the power spectrum of the current noise induced in the resonator is

$$S_{I_{th}} = \frac{S_{V_{th}}}{|Z_k|^2} = \frac{4k_B T_0 \omega_k}{Q_k L_k} \left(\frac{\omega^2 - \omega_k^2}{(\omega^2 - \omega_k^2)^2 + (\omega_k^2/ \omega_Q)^2}\right). \quad (3)$$

The Lorentzian shape of the spectrum is determined by the feedback-reduced quality factor $Q'_k$, while the prefactor is determined by the intrinsic quality factor $Q_k$. Integration of
Eq. (3) over frequency yields the total mean square current noise associated with mode \( k \):

\[
\langle I_k^2 \rangle = \frac{k_B T_0}{T_k} \frac{Q_k'}{Q_k} = \frac{k_B T_0}{L_k} \frac{1}{1 + g_k},
\]

so that, similarly to the quality factor, the effective temperature of the mode under feedback cooling is reduced to

\[
T_k = \frac{T_0}{1 + g_k}.
\]

As the temperature reduction is determined by the \( Q \) reduction, the larger is the initial quality factor, the larger is the achievable feedback cooling.

We can also rewrite Eq. (3) in terms of the mode temperature instead of the bath temperature:

\[
S_{I_n} = \frac{4k_B T_n \omega_k}{Q_k L_k} \omega^2 \left( \omega^2 - \omega_k^2 \right)^2 + \left( \frac{\omega_k \omega}{Q_k} \right)^2.
\]

This is precisely the expected power spectrum of a passive resonator with quality factor \( Q_k' \) at thermal equilibrium at temperature \( T_k \). Actually, the resonator is not at thermal equilibrium at \( T_k \), but is rather in a dynamical equilibrium between the thermal bath at \( T_0 \), and the measuring-feedback system, which acts as a very low temperature bath.

In the above analysis we have neglected the backaction and additive noise of the SQUID amplifier, which are expected to set a lower limit to the cooling efficiency. If the SQUID noise is taken into account, with the further simplified assumption of uncorrelated noise sources, the following refined expression of the mode temperature can be derived from the model in Fig. 2:

\[
T_k = \frac{1}{1 + g_k} \left( T_0 + \frac{Q_k S_{V_n}}{4k_B \omega_k L_k} \right) + \frac{g_k^2}{1 + g_k} \frac{\omega_k L_k}{Q_k} S_{I_n},
\]

where \( S_{I_n} \) is the spectral density of the SQUID measurement noise \( I_n \), fed back by the cooling loop, and \( S_{V_n} \) is the spectral density of the SQUID back-action noise \( V_n \). For our present setup, the backaction contribution is almost negligible with respect to thermal noise. According to Eq. (7), there is an optimum value of \( g_k \), for which the temperature achieves a minimum. This minimum achievable temperature is slightly dependent on the considered mode, and is of order 40 \( \mu K \) for our present setup.

We point out that feedback cooling of the modes does not improve the sensitivity of the system as GW detector. In fact, the cooling is due to a modification of the effective response of the system to any kind of excitation. Therefore, it suppresses in the same way both the thermal noise and the external signal originated by an impinging GW.

To test the feedback cooling technique, we modified the standard operating conditions of the AURIGA detector, and set four different values of the feedback cooling gain \( D \), corresponding to four values of the relative feedback damping \( g_k \). For each setting, we measured the power spectral density of the current detected by the SQUID sensor, and averaged for a time of roughly 1 h. The four spectra are shown in Fig. 3. The overall power spectral density for a given setting can be accurately fitted by a proper combination of three Lorentzian curves, one for each mode. In fact, differently from previous experiments focused on a single mode, we simultaneously cool all three modes. The three-mode fitting curve is also shown in Fig. 3, superimposed on the corresponding noise spectrum. For \( \omega \approx \omega_k \), the fitting function can be approximated by the single-mode expression Eq. (6), from which we can infer the effective temperature of each mode \( T_k \). The experimental values of \( T_k \) as function of the damping ratio \( \left( 1 + g_k \right)^{-1} \) are shown in Fig. 4. For a given feedback setting, different values of \( g_k \) are associated to the three modes, because the intrinsic resistances \( R_k \) are in general different. For instance, in our measurements \( g_1 \) ranges from 190 to 2000 and \( g_3 \) ranges from 2200 to 30 000. The values of \( g_k \) are still small enough to make almost negligible the effect of the SQUID noise. Therefore, we expect the effective temperatures to follow the simple behavior described by Eq. (5). The straight line in Fig. 4 represents \( T_k \) for all 3 modes, calculated using Eq. (5) without free parameters, with \( T_0 \) fixed to the thermodynamic temperature of the bath, \( T_0 = 4.2 \) K. The data are in good agreement with the predictions of the model, even well below 1 mK. The lowest achieved temperature are \( T_1 = 2.0 \) mK, \( T_2 = 0.17 \) mK, and \( T_3 = 0.20 \) mK. The lowest temperatures for modes 2 and 3 correspond to an average occupation number \( \langle N_k \rangle = k_B T_k / \hbar \omega_k = 4000 \).

These results represent an improvement by more than 1 order of magnitude with respect to the lowest temperature reported in literature for actively cooled macroscopic mechanical resonators [6], and only experiments per-
formed at much higher frequency [11] have achieved lower occupation number. Although in our experiment we actively cool electromechanical normal modes rather than purely mechanical modes, we believe that the comparison is meaningful, as all three normal modes collect a significant fraction of the energy of the two mechanical resonators. For instance, from the detector calibration we estimate that the mode 2, the coolest one, collects about 36% of the energy deposited by an impulsive excitation in the bar resonator. Our cooling results are particularly relevant because of the enormous size of the elements of our system with respect to previously analyzed micromechanical systems. This overturns the common belief that the cooling should be easier for lighter resonators. To explain this apparent paradox, we observe that the relevant parameters in determining the cooling capability are the displacement sensitivity and the intrinsic quality factor of the resonator, which sets the potential $Q$-reduction ratio. In our case, the latter is 1 order of magnitude higher than that usually achieved by micromechanical resonators. In fact, according to a well-established empirical rule, the quality factor of a mechanical resonator scales roughly with the volume to surface ratio, suggesting a limiting factor in the surface dissipation mechanisms [18]. As a consequence, large resonators made of high $Q$ material, with linear dimensions in the 1 cm–1 m range, can easily reach quality factor as large as $10^6$–$10^7$, whereas achieving $Q > 10^5$ in micron-sized resonators has as yet proven difficult [19]. We also notice that a very large quality factor is a necessary condition for the observation of any macroscopical quantum behavior. In fact, the time scale $\tau$ for decoherence in a resonator with quality factor $Q$ and temperature $T$ is proportional to the ratio $Q/T$ [7,20].

Significant improvements with respect to the results presented here are expected by performing a dedicated experiment on a midscale (0.01–1 kg) resonator at subkelvin bath temperature. Conventional dilution refrigerators can be used to cool kg-scale masses down to 10 mK [21]. At the same time, resonators made of very low dissipation material, like silicon or sapphire, can reach quality factors as large as $10^9$ [22]. Moreover, our capacitive-SQUID measurement system, similarly to SET-based ones [11], is naturally compatible with ultralow temperatures, due to the very low power dissipation, of order $10^{-10}$ W for typical dc SQUIDs. Eventually, the maximum cooling ratio will be limited by the SQUID sensitivity, according to Eq. (7). As state-of-art devices can approach the quantum limit [23–25], resonator temperatures lower than 1 $\mu$K are achievable at kHz frequency, corresponding to a single-digit occupation number of the quantum oscillator.

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*vinante@science.unitn.it