Reexamination of the Power Spectrum in De Sitter Inflation

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We find that the amplitude of quantum fluctuations of the invariant de Sitter vacuum coincides exactly with that of the vacuum of a comoving observer for a massless scalar (inflaton) field. We propose redefining the actual physical power spectrum as the difference between the amplitudes of the above vacua. An inertial particle detector continues to observe the Gibbons-Hawking temperature. However, although the resulting power spectrum is still scale-free, its amplitude can be drastically reduced since now, instead of the Hubble's scale at the inflationary period, it is determined by the square of the mass of the inflaton fluctuation field.

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The prediction of a nearly "scale-free" spectrum of density perturbations is commonly considered as a crucial prediction of inflationary cosmology [1]. Departures from homogeneity arise then as quantum fluctuations, ϕ, of the scalar inflaton field that drives inflation [2] (see also [3]). This prediction explains the power spectrum of the galaxy distribution and has also been successfully confirmed by high precision measurements [4] of the anisotropies in the cosmic microwave background. The amplitude of the spectrum was predicted to be proportional to the square of the Hubble constant during inflation

$$\Delta^2_\phi(k) \approx \hbar H^2, \tag{1}$$

although the precise estimate depends on the details of particular models. The resulting amplitude for GUT-scale inflation turned out to be several orders of magnitude too large, or required fine-tuning for model parameters, to account for the observed $\delta\rho/\rho \sim 10^{-4} - 10^{-5}$ and is still a rather elusive problem.

A simple argument that gives the above amplitude estimate comes from the Gibbons-Hawking radiation effect. As measured by a particle detector on a geodesic, the invariant vacuum state $|0_{\rm dS}\rangle$ in de Sitter space [5] has a nonzero temperature, the Gibbons-Hawking temperature $T_{\rm GH} = \frac{H\hbar}{2\pi}$ [6], where H is the Hubble constant of the exponentially expanding de Sitter universe

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2, \tag{2}$$

with $a(t) = e^{Ht}$. A comoving observer detects a thermal bath of radiation at temperature $T_{\rm GH}$ and the associated amplitude of thermal fluctuations accounts for (1).

However, in deriving the Gibbons-Hawking effect, one implicitly considers two vacuum states. In addition to the globally defined de Sitter vacuum, we have the local vacuum $|0_C\rangle$ associated with a freely falling, or comoving, observer at a given spatial point $\vec{x}$. The comoving observer perceives the de Sitter vacuum as a thermal bath of particles, with respect to the $|0_C\rangle$ vacuum. The fundamental argument underlying this result, as first explained for a general $a(t)$ in [7] (see also [8,9]), is that the modes defined in the two different vacua are related by a superposition of positive and negative frequencies and the corresponding creation and annihilation operators by a Bogoliubov transformation. In the present case, the positive-frequency modes defining the comoving vacuum u_i^C cannot be expressed in terms of the purely positive-frequency modes $u_j^{\rm dS}$ defining the de Sitter vacuum. It is just the comparison of one set of modes with respect to the other set that precludes the physical equivalence of both vacua, and the existence of an horizon for the comoving observer is then responsible for the exact thermal behavior of $|0_{\rm dS}\rangle$ in the Fock space of $|0_C\rangle$. The fundamental role of both vacua can be nicely displayed in terms of two-point functions. The standard formula for the expectation value of the particle number operator in terms of Bogoliubov coefficients [7] can be rewritten, when particularized to de Sitter space and the inhomogeneous scalar field ϕ, as follows [10]

$$\langle 0_{\rm dS}|N_i^C|0_{\rm dS}\rangle = \sum_j |\beta_{ij}|^2 = \frac{1}{\hbar}\int_\Sigma d\Sigma_1^\mu d\Sigma_2^\nu (u_i^C(x_1)\overleftrightarrow{\partial}_\mu)(u_i^{C*}(x_2)\overleftrightarrow{\partial}_\nu) \times [\langle 0_{\rm dS}|\phi(x_1)\phi(x_2)|0_{\rm dS}\rangle - \langle 0_C|\phi(x_1)\phi(x_2)|0_C\rangle], \tag{3}$$

where Σ is a Cauchy hypersurface. Explicit evaluation of the above expressions, either via Bogolubov coefficients [11] or two-point functions, reproduces the Planckian spectrum. The physical idea in the latter method is that it is just the difference between the correlations of the de Sitter vacuum and those of the comoving vacuum that produces the relevant observables. Similarly, in black hole emission [12], the difference between the two-point function for the "in" vacuum, defined at the remote past before gravitational collapse, and that for the "out" vacuum, defined at future infinity, is at the heart of Hawking radiation [10]. This idea can be reinforced by deriving the Gibbons-Hawking effect in terms of the Unruh particle detector [13]. The rate of the response function of an inertial detector in de Sitter space, with trajectory $x^\mu = x^\mu(\tau)$, is given by

$$\dot{F}(w) = \int_{-\infty}^{+\infty} d\Delta\tau e^{-iw\Delta\tau} \langle 0_{\rm dS}|\phi(x(\tau))\phi(x(\tau+\Delta\tau))|0_{\rm dS}\rangle, \tag{4}$$

which reproduces, via a detailed balance argument, the expected thermal result at the temperature $T_{\rm GH}$ [6]. Since the response function of the comoving detector vanishes in the comoving vacuum, using $i\epsilon$ prescription,

$$\int_{-\infty}^{+\infty} d\Delta\tau e^{-iw\Delta\tau} \langle 0_C|\phi(x(\tau))\phi(x(\tau+\Delta\tau))|0_C\rangle = 0, \tag{5}$$

one can, equivalently, compute the rate (4) by subtracting the corresponding two-point function of the comoving observer [14]

$$\dot{F}(w) = \int_{-\infty}^{+\infty} d\Delta\tau e^{-iw\Delta\tau} [\langle 0_{\rm dS}|\phi(x(\tau))\phi(x(\tau+\Delta\tau))|0_{\rm dS}\rangle - \langle 0_C|\phi(x(\tau))\phi(x(\tau+\Delta\tau))|0_C\rangle]. \tag{6}$$

The $i\epsilon$ regularization prescription of the Wightman function in (4) can be replaced, as a mathematical identity, by the subtraction of the two-point function for the comoving vacuum. Note that the integrand in (6) is now a smooth function as a consequence of the Hadamard condition for the two-point functions [15] and there is no need for the $i\epsilon$ prescription. Expression (6) shows again that the detector responds to the relative correlations between the quantum state and the vacuum of the comoving observer.

Having in mind all the above, we find it natural to propose that, to properly quantify the amplitude of quantum fluctuations, one should compare the amplitude of the modes $u_i^{\rm dS}$ of the invariant de Sitter vacuum with respect to the amplitude of the modes u_j^C of a comoving observer. This leads us to replace the standard definition of the power spectrum [3,16–18]

$$\int_0^\infty \frac{dk}{k}\Delta_\phi^2(k,t;{\rm dS}) = \langle 0_{\rm dS}|\phi(t,\vec{x})\phi(t,\vec{x})|0_{\rm dS}\rangle, \tag{7}$$

by the following

$$\int_0^\infty \frac{dk}{k}\Delta_\phi^2(k,t) \equiv \langle 0_{\rm dS}|\phi(t,\vec{x})\phi(t,\vec{x})|0_{\rm dS}\rangle - \langle 0_C|\phi(t,\vec{x})\phi(t,\vec{x})|0_C\rangle. \tag{8}$$

Note that, in the well-established Casimir effect between two conducting plates, it is also the difference between the formal vacuum energy density of the global quantum state with suitable boundary conditions on the plates, and the corresponding formal vacuum energy density of a (comoving) inertial observer at the same point in the absence of the plates, that is the physically relevant quantity. This subtle point of quantum field theory in nontrivial backgrounds, namely, the need to compare two quantum vacuum states, which has been checked experimentally in the Casimir effect, will be explored in this Letter in the de Sitter inflation.

An advantage of the new definition (8) is that its right-hand side is again a smooth function as a consequence of the Hadamard condition, which insures that the two-point functions in both the de Sitter and the comoving vacuum states have the same divergent parts. If we expand the right-hand side of (8) in modes, the resulting integral is finite, and no further renormalization is needed for $\Delta_\phi^2(k,t)$. Nevertheless, it should be clear that the reason for subtracting the amplitude of the comoving observer is more fundamental than simply to bypass the divergence of the two-point functions at the coincident point. The subtraction would be natural even if there were no divergences. With the standard definition (7), the right-hand side is formally divergent, implying that renormalization may play an important role in the evaluation of the physical power spectrum, as suggested and studied in [19], where adiabatic subtraction was used to write the renormalized two-point function or dispersion as a finite integral over modes.

Let us consider a minimally coupled scalar field in de Sitter space with $[\Box - (m/\hbar)^2]\phi(x) = 0$, where ϕ can be thought of as the quantum fluctuation of the inflaton field, $\phi_0(t) + \phi(x)$, and m is the mass of $\phi(x)$. The normalized modes $u_{\vec{k}}^{\rm dS}(\vec{x},t)$ for the invariant de Sitter vacuum are

$$u_{\vec{k}}^{\rm dS}(\vec{x},t) = \frac{1}{\sqrt{2(2\pi)^3 a(t)^3}} h_k(t) e^{i\vec{k}\vec{x}}, \tag{9}$$

$$h_k(t) = \sqrt{\frac{\pi}{2H}} H_n^{(1)}(kH^{-1}\exp(-Ht)), \tag{10}$$

where $n = \sqrt{9/4 - m^2/H^2\hbar^2}$ is the index of the Hankel function. Therefore, the amplitude of quantum fluctuations is given as a sum in modes

$$\langle 0_{\rm dS}|\phi(t,\vec{x})\phi(t,\vec{x})|0_{\rm dS}\rangle = \hbar(4\pi^2 a(t)^3)^{-1} \int_0^\infty |h_k(t)|^2 k^2 dk, \tag{11}$$

and the standard power spectrum is given by

$$\Delta^2_\phi(k,t;\mathrm{dS}) = \hbar(4\pi^2 a(t)^3)^{-1} k^3 |h_k(t)|^2. \quad (12)$$

Evaluated in terms of the physical comoving wave vector $\bar{k} = k/a(t)$, the amplitude behaves as in Minkowski space for very large $\bar{k}$, but around the exit from the Hubble horizon $\bar{k} \approx H$, and for $m \ll H\hbar$, one gets the usual nearly scale-free spectrum

$$\Delta^2_\phi(\bar{k};\mathrm{dS}) = \frac{\hbar H^2}{8\pi}|H_n^{(1)}(\bar{k}H^{-1})|^2 \approx \frac{\hbar H^2}{2\pi}. \quad (13)$$

Let us now study the amplitude of the comoving modes at a given spatial point $\vec{x}$. To this end, it is convenient to introduce static spherical coordinates

$$ds^2 = -(1-H^2\tilde{r}^2)d\tilde{t}^2 + \frac{d\tilde{r}^2}{1-H^2\tilde{r}^2} + \tilde{r}^2 d\Omega^2, \quad (14)$$

where, as usual, $d\vec{x}^2 = dr^2 + r^2 d\Omega^2$, and $\tilde{t} = -(2H)^{-1} \times \ln[e^{-2tH} - (rH)^2]$, $\tilde{r} = e^{tH} r$. We locate the origin of radial coordinates $\tilde{r} = 0$ at the location $\vec{x}$ of the arbitrary comoving observer. Note that, at $\tilde{r} = 0$, the new time coordinate $\tilde{t}$ coincides with the comoving time t, the metric takes the Minkowskian form, and the deviations from it are quadratic in $\tilde{r}$. In evaluating the amplitude of fluctuations $\langle 0_C|\phi(t,\vec{x})\phi(t,\vec{x})|0_C\rangle$ at the origin of coordinates, only the s-wave sector contributes, due to the regularity condition at $\tilde{r} = 0$. The s-modes are found to be

$$u_w^C = \frac{e^{-iw\tilde{t}}}{\sqrt{4\pi}}\frac{N_n(w)}{\tilde{r}}[P^{i(w/H)}_{n-1/2}(H\tilde{r}) - \alpha_n(w) Q^{i(w/H)}_{n-1/2}(H\tilde{r})] \quad (15)$$

where $P^\mu_\nu(z)$ and $Q^\mu_\nu(z)$ are generalized Legendre functions, $N_n(w)$ is a normalization constant, and $\alpha_n(w) = \frac{2}{\pi} \times \tan(\frac{\pi\mu_n}{2})$, where $\mu_n \equiv \frac{1}{2} + n + i\frac{w}{H}$, is a constant ensuring the regularity at $\tilde{r} = 0$. A major technical point is to compute the exact form of the normalization constant. Evaluating the scalar product at the future horizon, with tortoise coordinate $x \equiv H^{-1}\tanh^{-1}(\tilde{r}H)$, and taking into account the asymptotic oscillatory behavior of the functions P and Q at the horizon ($x \to +\infty$), namely, $P_n^{iw/H}(\tanh xH) \sim \Gamma(1 - iw/H)^{-1} e^{iwx}$, $Q_n^{iw/H}(\tanh xH) \sim A(w)e^{iwx} + B(w)e^{-iwx}$, with $A(w) = \frac{-2\pi i}{4\Gamma(1-iw/H)}/\tanh(\pi w/H)$, and $B(w) = \frac{-\pi}{4\Gamma(1-iw/H)}\frac{\coth\frac{\pi w}{2H}}{\sinh^2\frac{\pi w}{2H}}$, we find that $|N_n(w)|^2 \equiv \frac{1}{w}|\tilde{N}_n(w/H)|^2$, where $|\tilde{N}_n(w/H)|^2$ is the dimensionless function (where $z \equiv w/H$)

$$|\tilde{N}_n(z)|^2 = \frac{z}{4\sinh\pi z}\frac{1}{|1 + i\frac{\tan\frac{\pi\mu_n}{2}}{\tanh\pi z}|^2}. \quad (16)$$

Therefore, the form of the modes at the physically relevant point $\tilde{r} = 0$ can be written as

$$u_w^C(\tilde{r}=0) = e^{-iw\tilde{t}}\frac{\tilde{N}_n(w/H)}{\sqrt{4\pi w}} H\beta_n(w/H), \quad (17)$$

where (for $n \in \Re$)

$$\beta_n(z) = -\frac{2^{1/2-n}\sin(\pi\mu_n^*)\sin(\frac{\pi\mu_n}{2})}{\pi^2\cosh\pi z} \times \Gamma(\mu_n)\Big[z|\Gamma(\mu_{-n}/2)|^2 + 2\pi\Big(\frac{m^2}{\hbar^2 H^2} - 2\Big) \times \Im\Big(\frac{\Gamma(\mu_{-n})\,{}_2F_1(\frac{3}{2}-n, \frac{1}{2}-n+iz; 2+iz; -1)}{\Gamma(2+iz)\cos\frac{\pi\mu_n}{2}}\Big)\Big] \quad (18)$$

is a dimensionless function. With this, we obtain

$$\langle 0_C|\phi(t,\vec{x})\phi(t,\vec{x})|0_C\rangle = \frac{\hbar H^2}{4\pi}\int_{m/\hbar}^\infty \frac{dw}{w}|\tilde{N}_n|^2|\beta_n|^2. \quad (19)$$

Taking into account the relation, $w^2 = \bar{k}^2 + m^2\hbar^{-2}$, one gets the following spectrum of fluctuations

$$\Delta^2_\phi(\bar{k};C) = \hbar\frac{H^2}{4\pi}\frac{\bar{k}^2}{\bar{k}^2 + m^2\hbar^{-2}}|\tilde{N}_n(\bar{k}H^{-1})|^2|\beta_n(\bar{k}H^{-1})|^2]. \quad (20)$$

The amplitude of these fluctuations depends only on the physical comoving scale $\bar{k}$. The difference $\Delta^2_\phi(\bar{k};\mathrm{dS}) - \Delta^2_\phi(\bar{k};C)$, which is the proposed spectrum of this Letter, seems to be driven, at first sight, by H^2. However, explicit evaluation of the above formulas unravels a miraculous simplification of the right-hand side of (20) when the mass m goes to zero. In this case, $n = 3/2$, $\alpha_{3/2} = \frac{2i}{\pi}\tanh\frac{\pi w}{2H}$, $\beta_{3/2} = (1 + iw/H)/[\Gamma(1 - iw/H)\sinh^2\frac{\pi w}{2H}]$, and the normalization factor is

$$|\tilde{N}_{3/2}(w/H)|^2 = \frac{|\Gamma(1 - iw/H)|^2}{\pi}\sinh^2\frac{\pi w}{2H}. \quad (21)$$

We find that, irrespective of the scale $\bar{k}$, the amplitude of fluctuations is identical for both quantum states

$$\Delta^2_{\phi_{m=0}}(\bar{k};\mathrm{dS}) = \frac{\hbar H^2}{4\pi^2}\Big(1 + \frac{\bar{k}^2}{H^2}\Big) = \Delta^2_{\phi_{m=0}}(\bar{k};C). \quad (22)$$

This result has a major consequence since it implies that the proposed power spectrum,

$$\Delta^2_\phi(\bar{k}) \equiv \Delta^2_\phi(\bar{k};\mathrm{dS}) - \Delta^2_\phi(\bar{k};C), \quad (23)$$

is now driven by a different physical scale, namely, the mass of the scalar (inflaton) field, instead of the Hubble constant: $\Delta^2_\phi(\bar{k}) \propto m^2$ for small m^2.

Let us now estimate the behavior of the proposed power spectrum for the nonzero mass case. One immediately obtains that

$$\Delta^2_\phi(\bar{k}) = \hbar H^2\Big[\frac{1}{8\pi}|H_n^{(1)}(\bar{k}H^{-1})|^2 - \frac{1}{4\pi} \times \frac{\bar{k}^2}{\bar{k}^2 + m^2\hbar^{-2}}|\tilde{N}_n(\bar{k}H^{-1})|^2|\beta_n(\bar{k}H^{-1})|^2]\Big]. \quad (24)$$

This spectrum is still nearly scale free for $m^2/H^2\hbar^2 \ll 1$

and $\bar{k} \approx H$. In Table I, we compare the proposed power spectrum $\Delta^2_\phi(\bar{k})$ with the standard spectrum $\Delta^2_\phi(\bar{k}, \mathrm{dS})$ at $\bar{k} = H$ for different values of the inflaton mass. We observe that the amplitude of the proposed power spectrum scales with m^2, and the ratio with the conventional spectrum for $m^2/(H\hbar)^2 \leq 10^{-2}$ can be approximated by

$$\frac{\Delta^2_\phi(\bar{k})}{\Delta^2_\phi(\bar{k}, \mathrm{dS})}\bigg|_{\bar{k}=H} \approx 0.25\frac{m^2}{H^2\hbar^2}. \quad (25)$$

This shows that our proposal for subtracting the amplitude of the fluctuations of the comoving vacuum to define the power spectrum produces a drastic reduction of its amplitude provided that m^2 is chosen sufficiently small. It is worth noting that one of us [19] already found a similar behavior for the power spectrum on grounds of adiabatic regularization of the two-point function. The fact of getting similar numerical estimates from different approaches supports the robustness of this result.

In this Letter, we have explored an alternative definition for the power spectrum of quantum fluctuations in an inflationary de Sitter universe. An important result is that the amplitude of quantum fluctuations for the de Sitter invariant and the comoving vacuum states in de Sitter space coincide exactly in the massless case. This has major physical consequences. The proposed spectrum is no longer driven by the Hubble constant, but instead by the effective mass of the scalar field. This provides a natural way out of the problem of getting too large a magnitude for the amplitude of inflaton fluctuations since the magnitude can be automatically reduced by several orders of magnitude, and it merits further exploration. The Hubble constant during inflation may be larger without coming into contradiction with the observed amplitude of the CMB anisotropies.

Furthermore, as pointed out by one of us in [19], the vanishing amplitude for the case of $m = 0$ implies that the gauge-invariant tensor perturbations of the gravitational metric during exponential inflation may be 0. This is because in the Lifshitz gauge, the two polarization components of the gravitational tensor perturbations each satisfy the same equation as a minimally coupled scalar field with $m = 0$. Then, our proposal and the one in [19] imply that the tensor to scalar ratio may be smaller than previously predicted. The standard predictions for this ratio may soon come within the range of measurement.

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TABLE I. Ratio of the proposed power spectrum $\Delta^2_\phi(\bar{k})$ by the standard value $\Delta^2_\phi(\bar{k}, \mathrm{dS})$ at $\bar{k} = H$.

$\frac{m^2}{H^2\hbar^2}$	10^{-1}	10^{-3}	10^{-5}
$\frac{\Delta^2_\phi(\bar{k})}{\Delta^2_\phi(\bar{k},\mathrm{dS})}$	0.2212×10^{-1}	0.2525×10^{-3}	0.2529×10^{-5}

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