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Experimental Test of Fidelity Limits in Six-Photon Interferometry and of Rotational Invariance Properties of the Photonic Six-Qubit Entanglement Singlet State

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Quantum multiphoton interferometry has now reached the six-photon stage. Thus far, the observed fidelities of entangled states never reached $2/3$. We report a high fidelity (estimated at 88%) experiment in which six-qubit singlet correlations were observed. With such a high fidelity we are able to demonstrate the central property of these "singlet" correlations, their "rotational invariance," by performing a full set of measurements in three complementary polarization bases. The patterns are almost indistinguishable. The data reveal genuine six-photon entanglement. We also study several five-photon states, which result upon detection of one of the photons. Multiphoton singlet states survive some types of depolarization and are thus important in quantum communication schemes.

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Quantum information processing and communication relies on preparation, manipulation, and detection of specific highly nonclassical quantum states. Quantum superpositions, however, are very fragile and are easily destroyed by decoherence processes [1]. Such uncontrollable influences cause noise in the communication, or errors in a computation. Several strategies have been devised to cope with decoherence. For instance, if the qubit-environment interaction, no matter how strong, exhibits some symmetry, then there exist quantum states which are invariant under this interaction. These states are called decoherence-free (DF) states, and allow protection of quantum information [2,3].

Experimental efforts investigating features of DF systems have been carried out to demonstrate the properties of a specific two-qubit DF state [4], and the existence of three-qubit noiseless subsystems [5]. Bourennane *et al.* [6] produced the four-photon polarization-entangled state $|\Psi_4^-\rangle$, which is a generalization of the singlet, and demonstrated its invariance under general collective noise and experimentally demonstrated the immunity of a qubit encoded in this state. Cabello has theoretically constructed DF states formed by six qubits, which allow one to encode an arbitrary two-qubit state [7]. One of these is $|\Psi_6^-\rangle$. This state is, in the language of spin-$1/2$ representation of qubits, a singlet with respect to the total spin. Thus it is invariant under transformations which consist of identical unitary transformations of each individual constituent [2] according to $U^{\otimes 6}|\Psi_6^-\rangle = |\Psi_6^-\rangle$, where $U^{\otimes 6}$ denotes the tensor product of identical unitary operators, each acting on a different qubit.

The singlet state can be used for secure quantum multiparty cryptographic protocols such as the six-party secret sharing protocol [8,9]. However, all these beautiful properties can be of practical value to quantum information processing only if the state is produced with a high fidelity, and if the unique invariance properties are clearly observable.

With the paper of Lu *et al.* [10] multiphoton interferometry based on parametric down-conversion (PDC) reached the stage at which one can observe genuine six-photon interference. Several exciting experiments followed, with different experimental configurations. In the experiments various states were observed, however with not too high fidelities. The three-source noncollinear PDC experiment aimed at graph states [10], as well as two observations of the Dicke state, using multiple emission from single sources of collinear PDC [11,12], all achieved fidelities lower than 66%. This raises the question of whether we are doomed to face this problem in six-photon interferometry. In our recent experiment [13] we used a generalization of the blueprint of [14], and its realization [15]. That is, we used a single PDC source with noncollinear emissions. The multiphoton source (of up to six entangled photons) works by pulse pumping the crystal and extracting, via suitable filtering and coincidence registrations, the corresponding order processes. Multiphoton interference was observed behind a set of beam splitters [13]. Beam overlaps were entirely avoided, making the setup very robust. We showed a very good stability of the source and we observed two-, four-, and six-photon correlation oscillations with very high visibilities: 96%, 92%, and 84%, respectively. These values are only slightly lower than the theoretical ones [16] calculated for the pump and filter parameters used in the experiment. These properties of our new setup challenged us to test to what extent one can boost the fidelity of six-photon states with current state-of-the-art technology.

Here, we present a high fidelity experimental observation of the six-photon entangled $|\Psi_6^-\rangle$ singlet state and we also demonstrate its unique and basic property of rotational invariance of its correlations. The invariance tests of the experimental correlations are done by sequential measurements in three mutually complementary polarization bases [linear horizontal, vertical (H, V), linear diagonal, antidiagonal (D, A), and circular left, right (L, R)] at all six detection stations.

Another interesting feature of the state is that it reveals various interesting types of entanglement within its subsystems. This is studied here theoretically, and compared with the measurement data. Finally we present tests aimed at verifying the entanglement of the obtained state.

Within the second quantized description, the state component corresponding to the emission of six photons in a single type-II PDC process is proportional to $(a_H^\dagger b_V^\dagger + e^{i\phi} a_V^\dagger b_H^\dagger)^3|0\rangle$, where $a_H^\dagger$ $(b_V^\dagger)$ is the creation operator for one horizontal (vertical) photon in mode a (b), etc., ϕ is the phase difference between horizontal and vertical polarizations due to birefringence in the crystal, and $|0\rangle$ denotes the vacuum state. This is a good quantum optical description of the initial six-photon state, provided one collects the photons under conditions that allow the indistinguishability between separate two-photon emissions [17].

The third order PDC is fundamentally different than just a product of three entangled pairs. Because of the bosonic nature of photons the multifold emissions of indistinguishable photons of the same polarization are more likely than those with photons of orthogonal polarization.

Our objective is to generate and characterize the invariant six-qubit polarization-entangled state given by the following superposition of a six-qubit Greenberger-Horne-Zeilinger (GHZ) state and two products of three-qubit W states:

$$|\Psi_6^-\rangle = \frac{1}{\sqrt{2}}|\mathrm{GHZ}_6^-\rangle + \frac{1}{2}(|\bar{W}_3\rangle|W_3\rangle - |W_3\rangle|\bar{W}_3\rangle). \quad (1)$$

The GHZ state is here defined as $|\mathrm{GHZ}_6^-\rangle = \frac{1}{\sqrt{2}}(|HHHVVV\rangle - |VVVHHH\rangle)$, and the W state is defined as $|W_3\rangle = \frac{1}{\sqrt{3}}(|HHV\rangle + |HVH\rangle + |VHH\rangle)$. $|\bar{W}_3\rangle$ is the spin-flipped $|W_3\rangle$, and H and V denote horizontal and vertical polarization, respectively. This state is obtained from the third order emission of the PDC process with the phase $\phi = \pi$. The emitted photons are beam split into six modes, and via coincidence measurements we select the terms with one photon in each mode. It is easy to see that if one moves into the spin description of the polarization variables, the state is a singlet (total spin equal to zero) of a composite system consisting of six spin-1/2 particles.

In our experiment we use a setup previously tested for its very high stability, and for high visibilities of multiphoton interference of various types. For technical details of the setup please consult [13]. The schematic of the setup is given Fig. 1.

Figure 2(a) shows experimentally estimated probabilities to obtain each of the 64 possible sixfold coincidences with one photon detection in each spatial mode, measuring all qubits in (H,V) basis. The peaks are in very good agreement with theory: half of the detected sixfold coincidences are to be found as $HHHVVV$ and $VVVHHH$, and the other half should be evenly distributed among the remaining events with three H and three V detections. This is a clear effect of the bosonic interference (stimulated emission) in the beta barium borate (BBO) crystal giving higher probabilities for emission of indistinguishable photons.

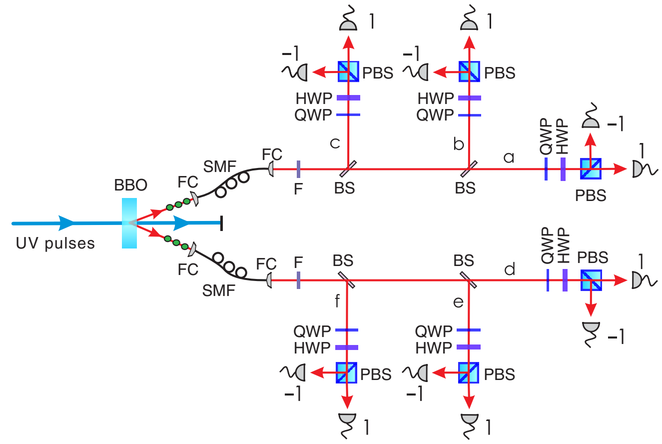

FIG. 1 (color online). Experimental setup for generating and analyzing the six-photon polarization-entangled state. The six photons are created in third order PDC processes in a 2 mm thick beta barium borate (BBO) pumped by UV pulses. The intersections of the two cones obtained in noncollinear type-II PDC are coupled, with the help of fiber couplers (FC), to single mode fibers (SMFs) wound in polarization controllers. Narrow band ($\Delta\lambda = 3$ nm) interference filters (F) serve to remove spectral distinguishability. The coupled spatial modes are divided into three modes each by 50:50 beam splitters (BSs). Each mode can be analyzed in arbitrary basis using half wave and quarter wave plates (HWP and QWP) and a polarizing beam splitter (PBS). Simultaneous detections of six photons (two single photon detectors for each mode) are being recorded by a 12 channel coincidence counter.

The six-photon state $|\Psi_6^-\rangle$ is invariant under identical (unitary) transformations U in each mode. Experimentally this can be shown by using *identical* settings of all polarization analyzers: no matter what the setting is, the results should be similar. Our results for measurements in D (A), $|D\ (A)\rangle = [|H\rangle \pm |V\rangle]/\sqrt{2}$, and L (R), $|L\ (R)\rangle = [|H\rangle \pm i|V\rangle]/\sqrt{2})$, polarization bases are presented in Figs. 2(b) and 2(c). The invariance of the probabilities with respect to the joint changes of the measurement basis in all modes is clearly visible. We also clearly observe the small and uniform noise contribution in all three measured bases.

Another property of $|\Psi_6^-\rangle$ is that it exhibits perfect EPR correlations between measurement results in different modes. Experimentally we have obtained the following values of the correlation functions: $\langle\sigma_z^{\otimes 6}\rangle = -0.88 \pm 0.04$, $\langle\sigma_x^{\otimes 6}\rangle = -0.88 \pm 0.05$, and $\langle\sigma_y^{\otimes 6}\rangle = -0.87 \pm 0.04$, which are close to the theoretical value of -1. From these results and the approximation that our noise is white, we have estimated the fidelity of the observed state: $F = \langle\Psi_6^-|\rho_{\mathrm{exp}}|\Psi_6^-\rangle = 0.88 \pm 0.03$, where ρ_{exp} denotes

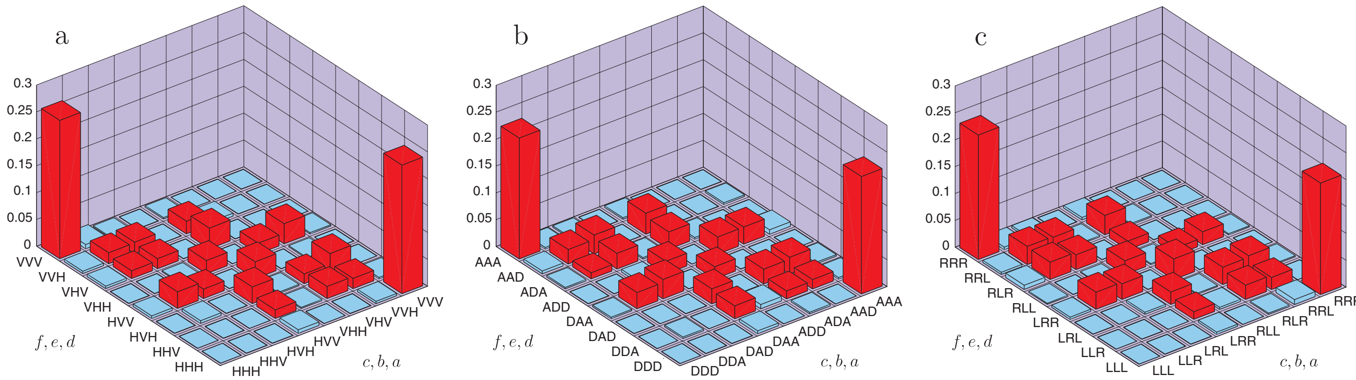

FIG. 2 (color online). Experimental results for the six-photon invariant state. All 64 (8×8) sixfold coincidence probabilities ($HHHHHH, \ldots, VVVVVV$), corresponding to detections of one photon in each mode in (H, V) basis (a), and similarly for (D, A) basis (b), and (L, R) basis (c) are shown. The values of the correlation functions are -0.88 ± 0.04, -0.88 ± 0.05, and -0.87 ± 0.04, respectively. The three sets of measurement results definitely reveal the invariance, which is a basic and unique property of the state. For the pure $|\Psi_6^-\rangle$ state the light blue (light gray) bars would be zero, and in our experiment their amplitudes are all of the order of the noise. The measurement time was about 140 h for each setting.

the experimental six-photon density given by $\rho_{\text{exp}} = p|\Psi_6^-\rangle\langle\Psi_6^-| + (1-p)\mathbb{1}^{\otimes 6}/2^6$, with $p \in [0, 1]$. The parameter p is calculated from the average of the three correlations presented above, as for each expectation value the following applies: $\langle\sigma_i^{\otimes 6}\rangle = \text{tr}(\sigma_i^{\otimes 6} \cdot \rho_{\text{exp}}) = -p$, $i = z, x, y$. The estimated fidelity clearly shows that our setup is able to produce correlations due to six-photon entanglement with unprecedented precision.

Note here, that taking into account that with the ratio of pump spectral width versus filter spectral width that characterized our experiments, which was 0.76, the expected maximal interference visibility is limited to 90%, see [13,16], and so is the strength of EPR correlations. Thus the obtained 88% allows us to claim that we are quite close to the achievable limits of these correlations and also of the fidelity.

Entanglement persistency (robustness) after a particle loss or projective measurement is an important feature in classifications of multipartite entanglement and in applications in quantum information [18]. In our experiment, conditioning on a detection of one of the photons in a specific state, we have obtained four different five-photon entangled states. In the diagonal basis the projection of the second qubit onto $|A\rangle_b$ leads to

$$ {}_b\langle A|\Psi_6^-\rangle = \frac{-1}{\sqrt{2}}|AADDD\rangle + \frac{1}{\sqrt{3}}|\Psi_2^+\rangle|W_3\rangle - \frac{1}{\sqrt{6}}|DD\rangle|\bar{W}_3\rangle, $$

where $|\Psi_2^+\rangle = \frac{1}{\sqrt{2}}(|DA\rangle + |AD\rangle)$ and the W states are defined as in Eq. (1) but with D and A substituted for H and V, respectively. A similar projection onto $|D\rangle_b$ results in

$$ {}_b\langle D|\Psi_6^-\rangle = \frac{1}{\sqrt{2}}|DDAAA\rangle - \frac{1}{\sqrt{3}}|\Psi_2^+\rangle|\bar{W}_3\rangle + \frac{1}{\sqrt{6}}|AA\rangle|W_3\rangle. $$

We have also performed such measurements related with the operator σ_z in mode b, which has the eigenstates $|H\rangle_b$ and $|V\rangle_b$. The projection onto $|V\rangle_b$ gives

$$ {}_b\langle V|\Psi_6^-\rangle = \frac{1}{\sqrt{2}}|\text{GHZ}_5^+\rangle - \frac{1}{\sqrt{6}}\left(|\Psi_2^+\rangle - \frac{1}{\sqrt{2}}|AA\rangle\right)|W_3\rangle - \frac{1}{\sqrt{6}}\left(|\Psi_2^+\rangle - \frac{1}{\sqrt{2}}|DD\rangle\right)|\bar{W}_3\rangle, $$

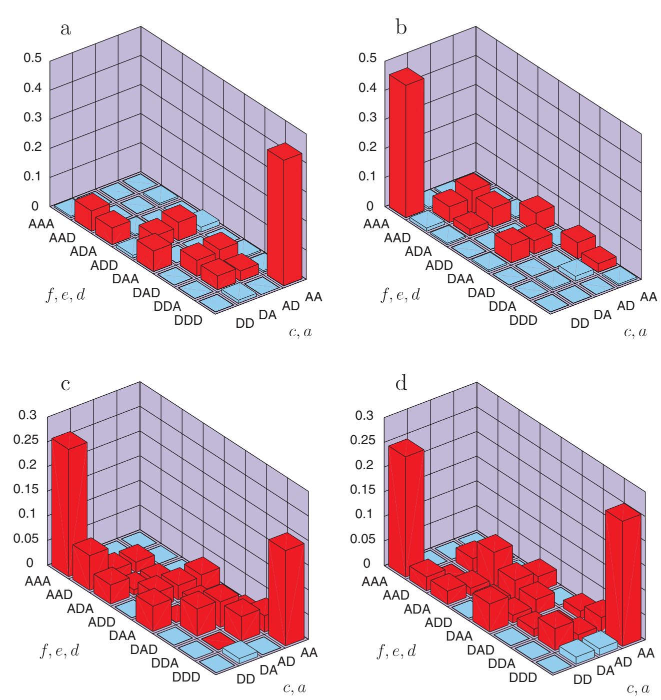

FIG. 3 (color online). Five-photon states from projective measurements. The 32 (8×4) possible fivefold coincidence probabilities ($DDDDD, \ldots, AAAAA$) are obtained through the projection of the photon in mode b onto $|A\rangle_b$ (a), $|D\rangle_b$ (b), $|V\rangle_b$ (c), and $|H\rangle_b$ (d), respectively. The remaining five photons are measured in the (D, A) basis.

and the projection onto $|H\rangle_b$ gives

$$ {}_b\langle H|\Psi_6^-\rangle = \frac{1}{\sqrt{2}}|\mathrm{GHZ}_5^-\rangle + \frac{1}{\sqrt{6}}\Big(|\Psi_2^+\rangle + \frac{1}{\sqrt{2}}|AA\rangle\Big)|W_3\rangle - \frac{1}{\sqrt{6}}\Big(|\Psi_2^+\rangle + \frac{1}{\sqrt{2}}|DD\rangle\Big)|\bar{W}_3\rangle, $$

where $|\mathrm{GHZ}_5^\pm\rangle = \frac{1}{\sqrt{2}}(|DDAAA\rangle \pm |AADDD\rangle)$.

Figure 3 shows measurement results in the (D, A) basis for these four five-photon conditional polarization states. In Figs. 3(a) and 3(b) the photon in mode b is projected onto $|A\rangle$ and $|D\rangle$, and we clearly see the terms $|AADDD\rangle$ and $|DDAAA\rangle$, respectively. Both these terms are evident in Figs. 3(c) and 3(d), where the projection was made onto the (H,V) basis. All these results are in agreement with theoretical predictions. This is an additional indication of the high fidelity of the singlet state obtainable with our setup.

$|\Psi_6^-\rangle$ is a six-qubit entangled state, meaning that each of its qubits is entangled with all the remaining ones. In order to show that our experimental correlations reveal six-qubit entanglement we use the entanglement witness method. An entanglement witness is an observable yielding a negative value only for entangled states, the most common being the maximum overlap witness, which is the best witness with respect to noise tolerance [19]. The maximum overlap witness optimized for $|\Psi_6^-\rangle$ has the form $\frac{2}{3}\mathbb{1}^{\otimes 6} - |\Psi_6^-\rangle\langle\Psi_6^-|$, where the factor 2/3 is the maximum overlap of $|\Psi_6^-\rangle$ with any biseparable state [20,21]. This witness detects six-partite entanglement with a noise tolerance around 34%, but it also demands a large number (183) of measurement settings. Since it is an experimentally very demanding task to perform all these measurements, we have developed a reduced witness that involves only three measurement settings. Our reduced witness $\mathcal{W}$ is given by

$$ \begin{aligned} \mathcal{W} = \frac{181}{576}\mathbb{1}^{\otimes 6} - \frac{1}{576}\sum_{i=x,y,z} (3\sigma_i^{\otimes 2}\mathbb{1}^{\otimes 4} + 3\sigma_i\mathbb{1}\sigma_i\mathbb{1}^{\otimes 3} \\ + 3\mathbb{1}\sigma_i^{\otimes 2}\mathbb{1}^{\otimes 3} + 3\mathbb{1}^{\otimes 3}\sigma_i^{\otimes 2}\mathbb{1} + 5\sigma_i^{\otimes 2}\mathbb{1}\sigma_i^{\otimes 2}\mathbb{1} \\ + 5\sigma_i\mathbb{1}\sigma_i^{\otimes 3}\mathbb{1} + 5\mathbb{1}\sigma_i^{\otimes 4}\mathbb{1} + 3\mathbb{1}^{\otimes 3}\sigma_i\mathbb{1}\sigma_i \\ + 5\sigma_i^{\otimes 2}\mathbb{1}\sigma_i\mathbb{1}\sigma_i + 5\sigma_i\mathbb{1}\sigma_i^{\otimes 2}\mathbb{1}\sigma_i + 5\mathbb{1}\sigma_i^{\otimes 3}\mathbb{1}\sigma_i \\ + 3\mathbb{1}^{\otimes 4}\sigma_i^{\otimes 2} + 5\sigma_i^{\otimes 2}\mathbb{1}^{\otimes 2}\sigma_i^{\otimes 2} + 5\sigma_i\mathbb{1}\sigma_i\mathbb{1}\sigma_i^{\otimes 2} \\ + 5\mathbb{1}\sigma_i^{\otimes 2}\mathbb{1}\sigma_i^{\otimes 2} - [\mathbb{1} \leftrightarrow \sigma_i]) - \frac{1}{64}\sum_{i=x,y,z}\sigma_i^{\otimes 6}, \end{aligned} \quad (2) $$

where $[\mathbb{1} \leftrightarrow \sigma_i]$ denotes the same terms as in the first sum but with $\mathbb{1}$ and σ_i interchanged. This is obtained from the maximum overlap witness as follows. First, the maximum overlap witness is decomposed into tensor products of Pauli and identity matrices. Next, only terms that are tensor products of σ_i with a fixed i and of identity matrices are selected (all terms that include products of at least two different Pauli matrices are deleted). Finally, the constant in front of $\mathbb{1}^{\otimes 6}$ in the first term of Eq. (2) is chosen to be the smallest possible such that all entangled states found by the reduced witness are also found by the maximal overlap witness. Our reduced witness detects six-partite entanglement of $|\Psi_6^-\rangle$ with a noise tolerance of 15%. The theoretical expectation value $\langle\mathcal{W}\rangle = -1/18 \approx -0.056$ and our experimental result is $\langle\mathcal{W}\rangle = -0.023 \pm 0.012$, showing entanglement with 2.0 standard deviations.

In summary, we have experimentally tested the current limits of the fidelity and the property of rotational invariance of the six-photon state $|\Psi_6^-\rangle$ produced by our setup. The state is indeed entangled, and various different entangled states can be obtained out of it with the use of projective measurements of one of the qubits. We would like to note that the interference contrast is high enough for our setup to be used in demonstrations of various six-party quantum informational applications (quantum reduction of communication complexity of some joint computational tasks, secret sharing, etc.). In forthcoming papers we shall present other states obtainable with our setup; see, e.g., [22].

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