Experiments on Elastic Cloaking in Thin Plates

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(Received 3 November 2011; published 3 January 2012)

Following a theoretical proposal [M. Farhat *et al.*, Phys. Rev. Lett. **103**, 024301 (2009)], we design, fabricate, and characterize a cloaking structure for elastic waves in 1 mm thin structured polymer plates. The cloak consists of 20 concentric rings of 16 different metamaterials, each being a tailored composite of polyvinyl chloride and polydimethylsiloxane. By using stroboscopic imaging with a camera from the direction normal to the plate, we record movies of the elastic waves for monochromatic plane-wave excitation. We observe good cloaking behavior for carrier frequencies in the range from 200 to 400 Hz (one octave), in good agreement with a complete continuum-mechanics numerical treatment. This system is thus ideally suited for demonstration experiments conveying the ideas of transformation optics.

In many situations in physics, one aims at steering or redirecting waves in a desired manner. When designing a structure to perform such a task, one generally encounters the "inverse problem" to find a structure that leads to the desired wave's behavior. The inverse problem is usually much harder to solve than the forward problem, for which one merely has to calculate the wave's behavior in a given structure via solving the underlying differential equations. Transformation optics [1,2] in combination with the concept of metamaterials has proven to be a valuable new tool for solving the inverse problem. Invisibility cloaking serves as a fascinating and demanding benchmark example. Importantly, however, the basic idea of transformation media goes well beyond the special field of electrodynamics at microwave, far-infrared, or optical frequencies. Yet, the detailed mathematics may well be distinct for different sorts of waves [3], for example, for tapered surface-plasmon-polariton waveguides [4], quantum-mechanical matter waves [5], bulk acoustic waves in solids [6], pressure waves in or at the surface of liquids [7–10], or elastic waves in thin plates or membranes [11,12].

Regarding the benchmark of invisibility cloaking, it is interesting to note that so far only very few successful experimental demonstrations for "free-space" cloaking have been published [4,8,9,13], for which cloaking works both in reflection and in transmission geometry. Notably, the largest measured fractional cloaking bandwidth of these four has been 10%–20% [4,8]. In contrast, the simplified so-called carpet or ground-plane cloak [14], which inhibits transmission by construction, has already been realized experimentally various times [10,15–22] and has reached a certain level of maturity. In this Letter, following the theoretical suggestion by Farhat *et al.* [12], we successfully design, fabricate, and experimentally characterize a free-space cloak for elastic waves in thin polymer plates at acoustic frequencies for what we believe is the first time. The fractional bandwidth of the cloak is at least one

octave—by far the largest value obtained for any free-space cloak to date.

Within the biharmonic approximation, the phase velocity c of elastic waves in plates or membranes that are thin compared to the wavelength $\lambda = 2\pi/k$ is given by [12]

$$c = \frac{\omega}{k} = k\sqrt{\frac{Eh^2}{12\rho(1-\nu^2)}},$$

where ω is the angular frequency and k the in-plane wave number of the wave, E the elastic or Young modulus of the plate, h its thickness, ρ its mass density, and ν its Poisson number. The main parameter to influence the waves is the material's Young modulus $E = E' + iE''$, which can be adjusted over a very large range. For example, we have measured the elastic modulus of polyvinyl chloride (PVC) as $E'_{\text{PVC}} = 2.4$ GPa and of polydimethylsiloxane (PDMS) as $E'_{\text{PDMS}} = 2$ MPa, which is more than 3 orders of magnitude smaller than that of PVC leading to a range of phase velocity which is unheard of in off-resonant optics. The other parameters are $E''_{\text{PVC}} = 48$ MPa, $\rho_{\text{PVC}} = 1.2$ kg/dm^3, $\nu_{\text{PVC}} = 0.4$, $E''_{\text{PDMS}} = 0.24$ MPa, $\rho_{\text{PDMS}} = 1.0$ kg/dm^3, and $\nu_{\text{PDMS}} = 0.5$. All parameters have been measured at room temperature and under 100 Hz excitation. The quoted nonzero imaginary parts refer to a phase-delayed response that leads to losses, which, however, are not due to nearby resonances in the polymer materials. Thus, the material parameters can be taken as frequency independent for the frequency range used in this Letter to very good approximation.

Intuitively, the idea of cloaking is to guide the wave around an object to let the wave emerge behind the object as if the object was not there. For a circular object and a circular cloak around it, the radial and the azimuthal phase-velocity components of the cloak need to be different. A larger azimuthal phase velocity allows the wave to make up for the additional propagation time due to the increased

path length it has to propagate over when making the detour around the object. This anisotropy needs to increase from the outer radius R_2 towards the inner radius $R_1 < R_2$ of the cloak. At the inner radius, the radial phase-velocity component must tend to zero to prevent any wave component from hitting the object. The physics is similar in electromagnetics [23].

More precisely, Farhat *et al.* [12] proposed a design of such cloak involving "only" 10 concentric rings of 6 different isotropic materials. The ring structure effectively leads to the desired anisotropy. However, thin plates composed of rings of 6 different actual materials are still relatively difficult to fabricate in practice. Inspired by optics, it would be advantageous to reduce the design to just 2 materials. In optics, vacuum or air is often chosen as one material. In contrast, for elastic waves, the tremendous impedance mismatch to air is prohibitive. Thus, we have chosen to replace the different actual elastic materials by metamaterials, each composed of the above two polymers PVC and PDMS. Upon changing the PVC volume filling fraction from 0% to 100%, the effective Young modulus can be tuned from the small PDMS value to the large PVC value given above. Such composites are easily realized by drilling holes into a PVC plate and filling them with PDMS.

Corresponding measurements (triangles) and calculations (dots) are depicted in Fig. 1. For each triangle, we have determined the effective elastic modulus by fabricating a thin slab of a metamaterial with this filling fraction and measuring its lowest elastic eigenfrequencies while fixing one end and leaving the other open. By comparison with theory [24], it is straightforward to extract the elastic modulus E'. We estimate an error in determining E' of about 10%. For filling fractions below about 40%, the slabs become too soft and no data could be taken. The dots in

Fig. 1 are the result of corresponding calculations using the commercial software package COMSOL MULTIPHYSICS.

Next, we map the continuous reduced cloak parameters according to Eq. (4) of Ref. [12] onto a structure composed of 20 homogeneous and locally isotropic rings. Using Eq. (3) of [12] and a genetic algorithm we compute the ring thicknesses and the elastic moduli that yield the best fit to Eq. (4) under the boundary condition that no elastic moduli above (below) that of PVC (PDMS) are possible. The obtained elastic moduli are given in the table in Fig. 2, the different ring thicknesses can be seen from the blueprint in the same figure. Actually, only 16 out of the

Ring	E′ (GPa)
20	2.22
19	0.249
18	2.4
17	0.231
16	2.1
15	0.171
14	2.4
13	0.165
12	2.4
11	0.133
10	1.84
9	0.063
8	1.99
7	0.048
6	2.26
5	0.03
4	2.4
3	0.009
2	2.4
1	0.002

Clamped region

FIG. 2 (color online). Top: Blueprint of the circular elastic cloak (only one-quarter is shown). White parts correspond to PVC, black parts to PDMS. The table on the right-hand side gives the elastic moduli (real parts) for the 16 different metamaterials (compare Fig. 1) in 20 concentric rings. The clamped region to be cloaked is shown in red, the surrounding region with homogeneous porosity ($f = 40\%$ PVC filling fraction) is highlighted in yellow. The plate thickness is $h = 1$ mm. Bottom: Oblique-view photograph of a cloak before filling with PDMS.
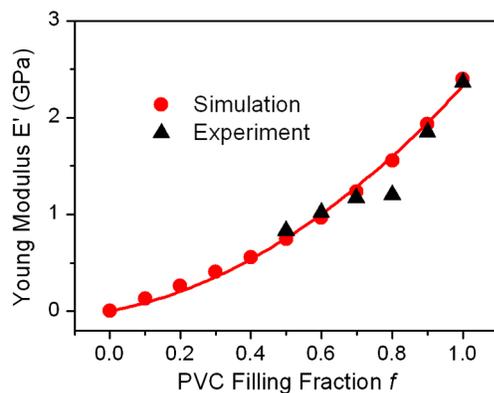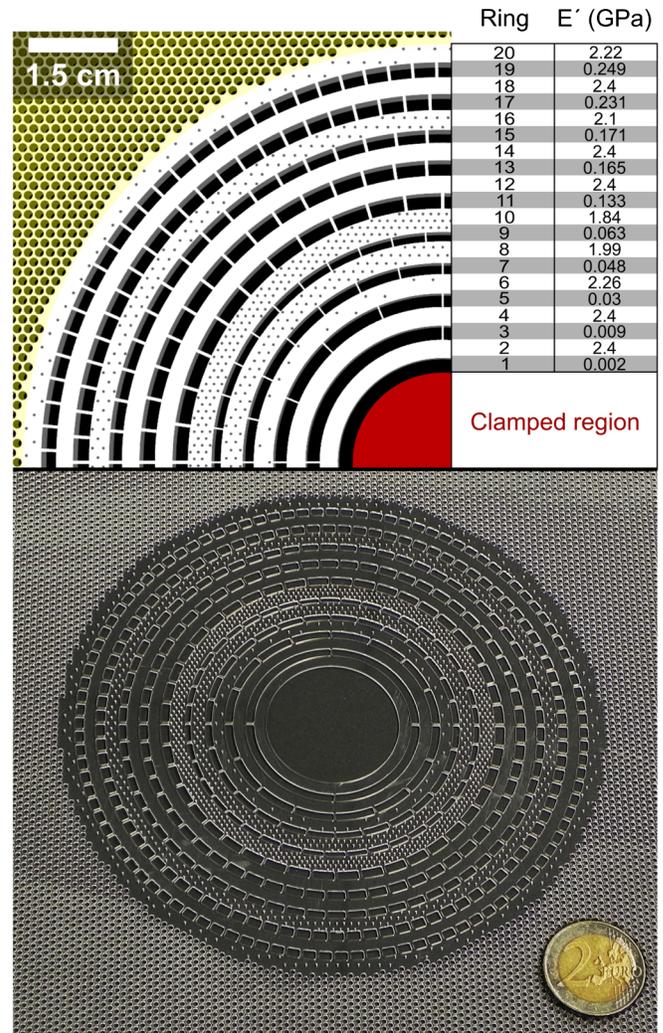

FIG. 1 (color online). Real part of the effective elastic Young modulus E' versus volume filling fraction f of polyvinyl chloride (PVC) in a $h = 1$ mm thin plate composed of PVC and polydimethylsiloxane (PDMS). The triangles are measured values, the dots are numerically calculated, and the solid curve is a fit to the numerical data under the boundary condition $E'(f = 0) = E'_{\text{PDMS}}$.

20 rings turn out to be significantly different in modulus. Finally, these elastic moduli are translated to local PVC filling fractions via the dependence $E'(f)$ shown in Fig. 1. The region around the cloak is also a metamaterial with a PVC filling fraction of 40%. This means that the entire plate (and not only the cloak) needs to be structured. A scheme and a photograph of the fabricated cloaking structure are shown in Fig. 2. Here, we have drilled holes and machined rings out of a 1 mm thin PVC plate. To keep a connected structure also for the third ring (see Fig. 2), we have added small bridges between the neighboring concentric rings that would otherwise be mechanically disconnected before the subsequent filling with PDMS. We expect that these few subwavelength-size necks do not influence the elastic wave propagation too much. The resulting mechanically robust structure is then painted in gray, which makes the entire plate appear homogeneous in the optical imaging experiments.

Plane waves in the rectangular plate are excited by loudspeakers at one end. The resulting behavior of the plate, which is stroboscopically illuminated with diffuse light from an array of light-emitting diodes, is recorded by a camera from the top. To enhance the image contrast, we subtract the temporal average gray level from all images. An important technicality lies in avoiding spurious reflections from the other end and from the two sides of the plate. We accomplish this by adding absorbing foam material that is gradually pushed against the plate from both sides, mimicking the perfectly matched layers used in many computer simulations. The quality of reflection suppression can easily be checked experimentally by letting a plane wave propagate in an homogeneous plate (not depicted). Spurious standing-wave components are an obvious indicator for imperfections. At frequencies in the range of 100 Hz and below, the used foam material no longer absorbs well and, hence, reflections obscure the results. Hence, we refrain from showing data within this frequency range in this Letter.

Figure 3 shows typical snapshots of a monochromatic plane wave (a) in a homogeneously structured plate with a circular region (as indicated in black) clamped to zero amplitude and (b) the same clamped region with the cloak around and the same surrounding. The four rows correspond to excitation frequencies of 200, 300, 400, and 450 Hz. Complete movies that give a much better overview are available via the Supplemental Material [25]. Clearly, the clamped region leads to substantial scattering of the wave, resulting in a shadow region behind the object and standing waves in front of it. Furthermore, the transmitted wave fronts are not plane waves but rather exhibit a curvature behind the scatterer. All three aspects are strongly suppressed in the presence of the cloak for all frequencies shown. While cloaking is nearly perfect for 200 Hz, increasing imperfections are visible with increasing frequency (also see quantification in Supplemental Material
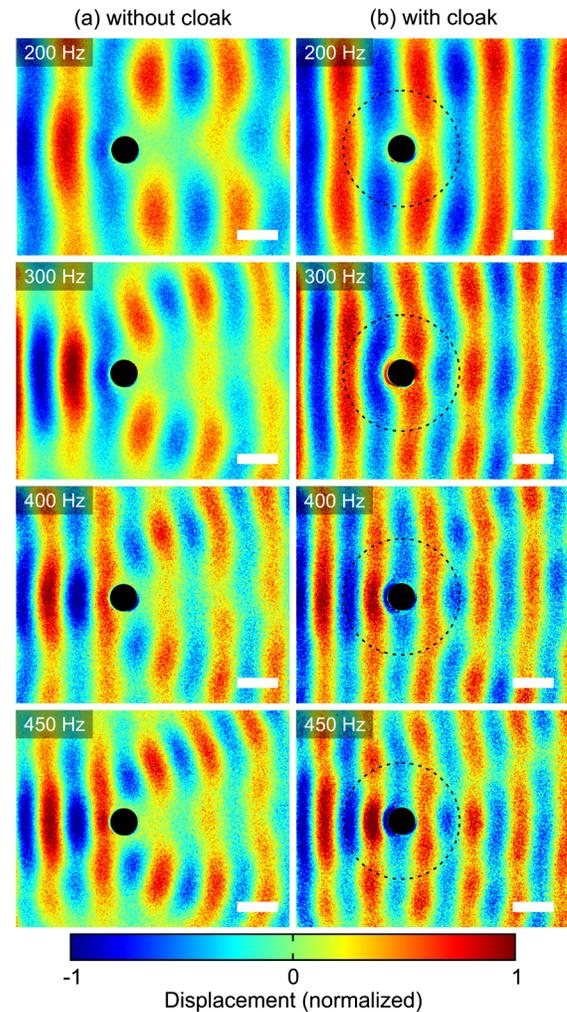

FIG. 3 (color online). Measured snapshots of thin-plate excursions upon injecting a monochromatic plane wave from the left-hand side. Reflections from the other three sides are suppressed by introducing adiabatically tapered absorber regions. The left column (a) corresponds to the homogeneous plate. The clamped region is shown in black (red in Fig. 2). The right column (b) corresponds to the scatterer plus cloak around. The dashed black circles illustrate the outer radius of the cloak. The four rows are for excitation frequencies of 200, 300, 400, and 450 Hz. Complete movies are available via the Supplemental Material [25]. The white scale bar in each image is 5 cm.

[25]). The same overall behavior is found in numerical calculations using the commercial software package COMSOL MULTIPHYSICS (see Fig. 4; for movies see Supplemental Material [25]). These calculations treat the plate in three dimensions and can be viewed as an exact numerical solution of the problem within continuum mechanics. In particular, longitudinal and shear deformations as well as rotational inertia are included. As usual the shear modulus G is connected with the Young modulus E via $G = E/[2(1 + \nu)]$. The cloak is treated by a set of concentric rings composed of locally homogeneous isotropic material. The corresponding densities, Poisson numbers,
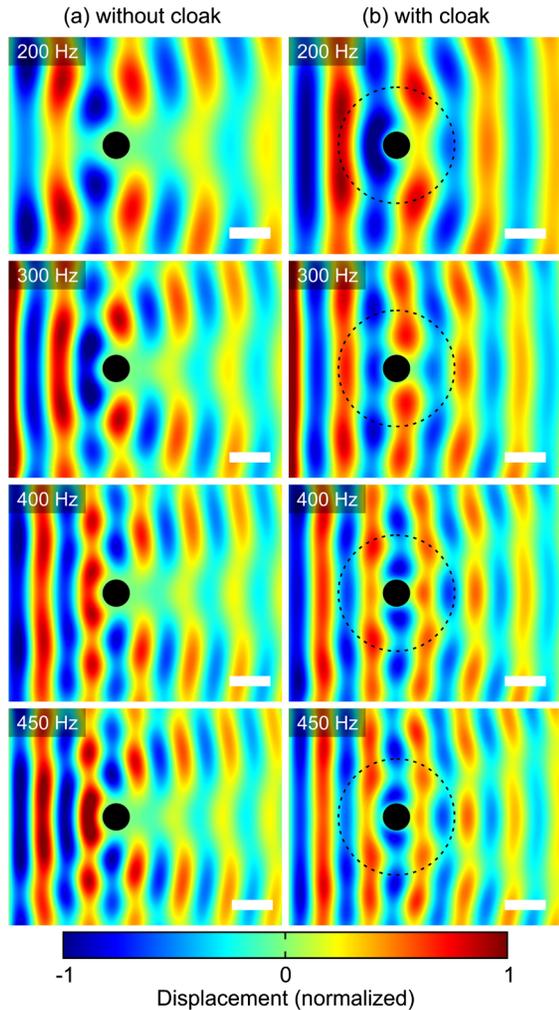
(a) without cloak   (b) with cloak

200 Hz   200 Hz

300 Hz   300 Hz

400 Hz   400 Hz

450 Hz   450 Hz

-1   0   1
Displacement (normalized)

FIG. 4 (color online). Three-dimensional numerical calculations corresponding to the experiments shown in Fig. 3 All parameters correspond to the experiment. Complete movies are available via the Supplemental Material [25].

and complex-valued Young moduli are the same as in the experiment (see above). This treatment goes beyond the biharmonic approximation used to design our cloak [12] and it also goes beyond the Mindlin-Reissner plate theory, both of which are effectively two dimensional [26]. Indeed, the biharmonic approximation neglects shear and torsional deformations and only becomes good in the limit of very long wavelengths compared to the relevant dimensions, e.g., plate thickness and thickness of the concentric rings. In Fig. 4, excellent cloaking is again found at 200 Hz frequency and the cloaking quality somewhat deteriorates towards higher frequencies. This experiment-theory comparison indicates that fabrication imperfections play only a very minor role in our experiments.

The observed dependence on frequency may be surprising considering the corresponding wavelength-to-size ratios. For example, for 400 Hz frequency, the wavelength in the homogeneous part of the plate is 6 cm (see Fig. 3). This

number has to be compared with the thickness of the concentric rings of 5.5 mm, as the distance between the holes within the rings is yet smaller. In previous optics experiments, cloaking has worked down to wavelengths that were even comparable to the lattice constant [13]. In the present elastic experiments, in contrast, imperfections already arise for wavelengths that are 11 times larger than the ring thickness. We assign this difference to the fact that the elastic cloak has been designed [12] using the biharmonic approximation, which neglects transverse shear deformation and rotational inertia [26,27], but which also leads to a simple coordinate-transformation invariant form. In optics, the underlying Maxwell equations are strictly coordinate-transformation invariant.

In conclusion, we have designed, fabricated, and experimentally characterized a broadband elastic free-space cloak in thin plates at acoustic frequencies. This solid-state system is macroscopic, very intuitive, and fairly inexpensive to make, and thus ideally suited for demonstration experiments. In addition, it is one of very few free-space cloaking systems that have successfully been realized experimentally thus far. Of these, our elastic cloak has by far the largest measured relative bandwidth (at least one octave). The elastic cloak can also serve as a model system for seismic cloaks that could protect sensitive infrastructure from earthquakes as discussed in [12].

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