Cosmological Relaxation of the Electroweak Scale

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A new class of solutions to the electroweak hierarchy problem is presented that does not require either weak-scale dynamics or anthropics. Dynamical evolution during the early Universe drives the Higgs boson mass to a value much smaller than the cutoff. The simplest model has the particle content of the standard model plus a QCD axion and an inflation sector. The highest cutoff achieved in any technically natural model is 10^8 GeV.

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Introduction.—In the 1970s, Wilson [1] had discovered that a fine-tuning seemed to be required of any field theory which completed the standard model Higgs sector, unless its new dynamics appeared at the scale of the Higgs mass. Since then, there have essentially been one and a half explanations proposed: dynamics and anthropics.

Dynamical solutions propose new physics at the electroweak scale which cuts off contributions to the quadratic term in the Higgs potential. Proposals include supersymmetry, compositeness for the Higgs boson (and its holographic dual), extra dimensions, or even quantum gravity at the electroweak scale [2–6]. While these scenarios all lead to a technically natural electroweak scale, collider and indirect constraints force these models into fine-tuned regions of their parameter spaces. Anthropics, on the other hand, allows for the tuning, but it assumes the existence of a multiverse. Its difficulty is in the inherent ambiguity in defining both probability distributions and observers.

We propose a new class of solutions to the hierarchy problem. The Lagrangian of these models is not tuned and yet has no new physics at the weak scale cutting off loops. In fact, the simplest model has no new physics at the weak scale at all. It is instead dynamical evolution of the Higgs mass in the early Universe that chooses an electroweak scale parametrically smaller than the cutoff of the theory. Our theories take advantage of the simple fact that the Higgs mass squared equal to zero, while not a special point in terms of symmetries, *is* a special point in terms of dynamics; namely, it is the point where the weak force spontaneously breaks and the theory enters a different phase [7]. It is this which chooses the weak scale, allowing it to be very close to zero. This mechanism takes some inspiration from Abbott's attempt to solve the cosmological constant problem [8].

Our models only make the weak scale technically natural [9], and we have not yet attempted to UV complete them for a fully natural theory, though there are promising directions [10–14]. Note that technical naturalness means a theory still contains small parameters, yet they are quantum mechanically stable, and therefore one can imagine field-theoretic UV completions. In addition, our models require large field excursions, far above the cutoff, and small couplings. We judge the success of our models by how far they are able to naturally raise the cutoff of the Higgs boson. Our simplest model can raise the cutoff to $\sim$1000 TeV, and we present a second model which can raise the cutoff up to $\sim 10^5$ TeV.

Minimal model.—In our simplest model, the particle content below the cutoff is just the standard model plus the QCD axion [15–17], with an unspecified inflation sector. Of course, by itself the QCD axion does not solve the hierarchy problem. However, the only changes we need to make to the normal axion model are to give the axion a very large (noncompact) field range, and a soft symmetry-breaking coupling to the Higgs boson.

The axion will have its usual periodic potential, but now extending over many periods for a total field range that is parametrically larger than the cutoff (and may be larger than the Planck scale), similar to recent inflation models such as axion monodromy [10–14]. The exact (discrete) shift symmetry of the axion potential is then softly broken by a small dimensionful coupling to the Higgs boson. This small coupling will set the weak scale and will be technically natural, making the weak scale technically natural and thus solving the hierarchy problem.

We add to the standard model Lagrangian the following terms:

$$(-M^2 + g\phi)|h|^2 + V(g\phi) + \frac{1}{32\pi^2}\frac{\phi}{f}\tilde{G}^{\mu\nu}G_{\mu\nu}, \qquad (1)$$

where M is the cutoff of the theory (where standard model loops are cutoff), h is the Higgs doublet, $G_{\mu\nu}$ is the QCD field strength (and $\tilde{G}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$), g is our dimensionful coupling, and we have neglected order one numbers. We have set the mass of the Higgs boson to be at the cutoff M so that it is natural. The field ϕ is like the QCD axion, but it can take on field values much larger than f. However, despite its noncompact nature it has all of the properties of the QCD axion with couplings set by f. Setting $g \to 0$, the Lagrangian has a shift symmetry $\phi \to \phi + 2\pi f$ (broken from a continuous shift symmetry by nonperturbative QCD effects). Thus, g can be treated as a spurion that breaks this symmetry entirely. This coupling can generate small potential terms for ϕ, and we take the potential with technically natural values by expanding in powers of $g\phi$. Nonperturbative effects of QCD produce an additional potential for ϕ, satisfying the discrete shift symmetry. Below the QCD scale, our potential becomes

$$(-M^2 + g\phi)|h|^2 + (gM^2\phi + g^2\phi^2 + \cdots) + \Lambda^4\cos(\phi/f), \qquad (2)$$

where the ellipsis represents terms higher order in $g\phi/M^2$, and thus we take the range of validity for ϕ in this effective field theory to be $\phi \lesssim M^2/g$. We have approximated the periodic potential generated by QCD as a cosine, but in fact the precise form will not affect our results. Of course Λ is very roughly set by QCD, but with important corrections that we will discuss below. Both g and Λ break symmetries, and it is technically natural for them to be much smaller than the cutoff. The parameters g and Λ are responsible for the smallness of the weak scale. This model plus inflation solves the hierarchy problem.

We will now examine the dynamics of this model in the early Universe. We take an initial value for ϕ such that the effective mass squared of the Higgs boson, m_h^2, is positive. During inflation, ϕ will slow roll, thereby scanning the physical Higgs mass. At some point in the ϕ potential, the quadratic term for the Higgs boson crosses zero and the Higgs boson develops a vacuum expectation value (VEV). As the Higgs VEV grows, the effective heights of the bumps, Λ^4, in the periodic potential grow. When the bumps are large enough, they become barriers which stop the rolling of ϕ shortly after m_h^2 crosses zero. This sets the Higgs mass to be naturally much smaller than the cutoff (see Fig. 1). Since it is the axion which is responsible for the dynamical relaxation of the weak scale, we call it the relaxion.

The (rel-)axion barrier height depends on the Higgs VEV through its dependence on quark masses [16]. When the Higgs VEV is near its standard model value, the potential barrier is approximately

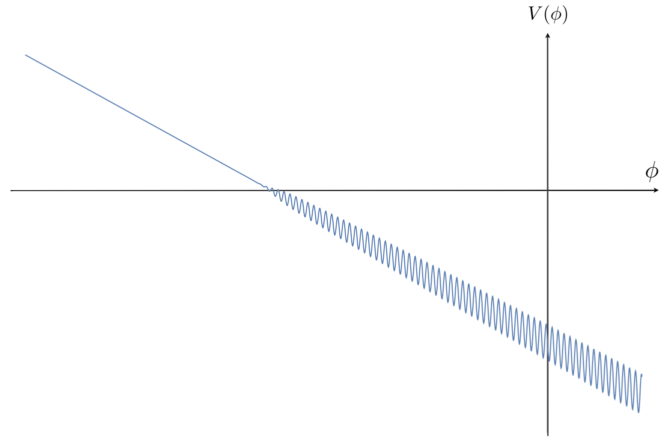

FIG. 1 (color online). Here is a characterization of the ϕ's potential in the region where the barriers begin to become important. This is the one-dimensional slice in the field space after the Higgs boson is integrated out, effectively setting it to its minimum. To the left, the Higgs VEV is essentially zero and is $\mathcal{O}(m_W)$ when the barriers become visible. The density of the barriers is greatly reduced for clarity.

$$\Lambda^4 \sim f_\pi^2 m_\pi^2 \qquad (3)$$

times the dimensionless ratios of the quark masses. Since m_π^2 changes linearly with the quark masses, it is proportional to the Higgs VEV. Therefore, Λ^4 grows linearly with the VEV [18].

During inflation, the relaxion must roll over an $\mathcal{O}(1)$ fraction of its full field range, $\sim(M^2/g)$, to naturally cross the critical point for the Higgs boson where $m_h^2 = 0$. Note that for the early Universe dynamics, one can consider the potential to be just $gM^2\phi$ or $g^2\phi^2$ since the field value for $\phi \sim (M^2/g)$ makes these equivalent. Our solution is insensitive to the initial condition for ϕ (as long as the Higgs boson starts with a positive mass squared) because ϕ is slow rolling due to Hubble friction. This places the slow-roll constraints on ϕ that $g < H_i$ and $g < (H_i^2 M_{\rm pl}/M^2)$, where H_i is the Hubble scale during inflation and $M_{\rm pl}$ is the reduced Planck mass. It is critical that ϕ is slowly rolling, not only because ϕ will reach a terminal velocity, $\dot{\phi} \simeq V'/H_i$, and thus be insensitive to initial conditions, but because it will keep ϕ's kinetic energy small allowing the onset of barriers to stop its evolution, regardless of its initial value. In the end, it will turn out that these constraints are trivially satisfied and superseded by stronger constraints below.

A requirement on inflation is that it lasts long enough for ϕ to scan the entire range. During N e-folds of inflation, ϕ changes by an amount $\Delta\phi \sim (\dot{\phi}/H_i)N \sim (V'_\phi/H_i^2)N \sim (gM^2/H_i^2)N$. Requiring that $\Delta\phi \gtrsim (M^2/g)$ gives the requirement on N,

$$N \gtrsim \frac{H_i^2}{g^2}. \qquad (4)$$

There are three conditions on the Hubble scale of inflation. First is that the vacuum energy during inflation is greater than the vacuum energy change along the ϕ potential, namely, M^4, so

$$H_i > \frac{M^2}{M_{\rm pl}} \quad \text{(vacuum energy)}. \tag{5}$$

The second constraint is the requirement that the Hubble scale during inflation is lower than the QCD scale (so the barriers form in the first place):

$$H_i < \Lambda_{\rm QCD} \quad \text{(barriers form)}, \tag{6}$$

where $\Lambda_{\rm QCD}$ is taken to be the scale where the instanton contributions to the axion potential are unsuppressed. We expect numerically that $\Lambda_{\rm QCD} \sim \Lambda$. Finally, a condition could be placed on the Hubble scale by requiring that ϕ's evolution be dominated by classical rolling (and not quantum fluctuations—similar to a constraint of $\delta\rho/\rho < 1$ in inflation) so that every inflated patch of the Universe makes it to the electroweak vacua

$$H_i < \frac{V'_\phi}{H_i^2} \to H_i < (gM^2)^{1/3} \qquad \text{(classical beats quantum)}. \tag{7}$$

We will see below that, while this constraint will be a bit stronger than the previous one, a certain variation of the model can avoid this constraint, in which case the previous one becomes the relevant one.

The slow rolling of ϕ stops when Λ has risen to the point where the slope of the barriers Λ^4/f matches the slope of the potential, gM^2. This occurs at [19]

$$gM^2 f \sim \Lambda^4. \tag{8}$$

From the three condition equations (5), (7), and (8), we have a constraint on the cutoff M:

$$M < \left(\frac{\Lambda^4 M_{\rm pl}^3}{f}\right)^{1/6} \sim 10^7\ \text{GeV} \times \left(\frac{10^9\ \text{GeV}}{f}\right)^{1/6}, \tag{9}$$

where we have scaled f by its lower bound of 10^9 GeV set by astrophysical constraints on the QCD axion (see, for example, Ref. [20]).

Note that in order to have a cutoff M above the weak scale, m_W, Eq. (8) requires $gf \ll m_W^2$. This implies that the effective step size of the Higgs mass from one minimum to the next is much smaller than the weak scale. So the barriers grow by a tiny fractional amount compared to $\Lambda_{\rm QCD}$ per step. Classically, ϕ stops rolling as soon as the slope of its potential changes sign. However, since $gf \ll m_W^2$, the slope of the first barrier after this point is exceedingly small, much smaller than Λ^4/f. Therefore, around this point, quantum fluctuations of ϕ will be relevant. The field ϕ will be distributed over many periods f (see Fig. 2), but in all of these the Higgs boson will have a weak-scale VEV. This quantum spreading is an oddity of the model. As the Universe inflates, different patches of the Universe will have a range of ϕ field values and a range of Higgs VEVs, but all around the weak scale. In future work, we will show it is possible to build models which land the full initial patch in a single vacuum, thus removing this feature of our solution [21].

At the end of inflation, part of the resulting ϕ range stops before the classical stopping point and is therefore classically unstable. This is because, during inflation, the relaxion's quantum fluctuations dominates its classical rolling on these "ledges" in the potential, and thus some tiny fraction of Hubble patches remain there until they can classically roll. The vast majority of Hubble patches find ϕ in vacua with varying potential barrier heights. Because of the small value of $gf \sim \Lambda^4/M^2$, barrier heights grow slowly, with subsequent minima increasing their barriers by $\sim\Lambda^4(\Lambda^4/(M^2 m_h^2))$ such that many barriers can be "walked over" via field fluctuations of the order of the Hubble scale. Eventually, most patches reach vacua where the barrier height does not allow quantum fluctuations to randomly walk over the barrier. Today, nearly all of the classically stable vacua have lifetimes exponentially larger than the age of the Universe. Therefore, the vast majority of the Hubble patches at the end of inflation are in vacua which last much longer than today's Hubble time. As a result of these multiple vacua, there will, in principle, be domain walls after reheating in the full initial patch of the

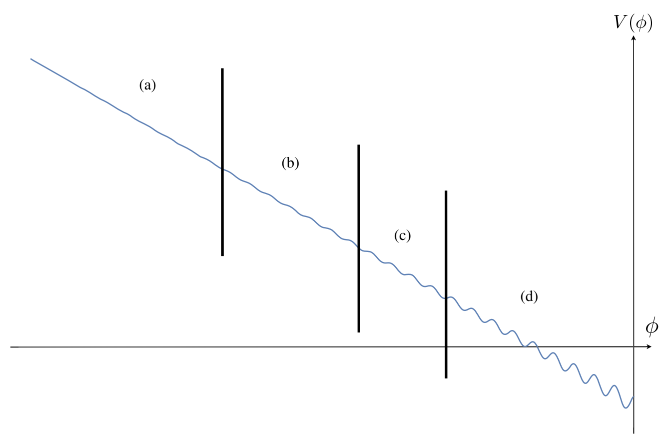

FIG. 2 (color online). A close-up of the region of ϕ's potential as the barriers appear. The evolution in these regions are (a) classical rolling dominated, (b) dominated by quantum fluctuations in the steps but classical rolling between steps, (c) classically stable, but quantum fluctuations or tunneling rates shorter than N e-folds, and (d) classically stable, quantum transition rates longer than both N e-folds and 10 Gyr. Again, for clarity, the potential is not to scale.

Universe. However, these domain walls will be spaced by distances much larger than our current Hubble size because we have much more than 60 e-folds of inflation in any one vacuum, and they are therefore unlikely to be observable.

We wish to avoid eternal inflation in our scenario because at least some part of the Universe would end up with a Higgs VEV above the weak scale. The decay rates to such vacua are exponentially suppressed but with a long enough period of inflation, some fraction of the Universe would end up there before reheating. Although this might naively seem like a very small part of the Universe, if we wish to avoid discussion of measures in eternal inflation, we must avoid this possibility. To do so, we can impose the constraint in Eq. (7), in which case the entire initial patch has the correct order electroweak scale at the end of inflation.

As noted above, even if we do not have eternal inflation in patches with large Higgs masses, we unfortunately cannot avoid ending up in a large range of vacua. Since all of these vacua have weak-scale Higgs VEVs, we call this a solution to the hierarchy problem. Of course, we have not solved the cosmological constant (CC) problem. This set of final vacua will all have different cosmological constants. If the solution to the CC problem is just tuning, then we must live in the one with the correct CC. This is just the usual tuning required for the CC problem, and not an additional tuning. Note that the other vacua with positive CCs will eternally inflate (as is our Universe, presumably), but in any case they will have a weak-scale Higgs VEV for a period that lasts much longer then 10 Gyr.

The model above is ruled out by the strong CP problem. Since ϕ is the QCD axion, its VEV determines the θ parameter in QCD. The relation Eq. (8) which determines where ϕ stops rolling predicts that the local minimum for ϕ is displaced from the minimum of the QCD part of the potential by $\mathcal{O}(f)$. Therefore, it generates $\theta \sim 1$. We found two solutions to this problem: (i) Potential barriers for ϕ arise from a new strong group, not QCD. (ii) The slope of the ϕ potential decreases dynamically after inflation. We discuss the latter solution below and the former in section NonQCD model. Of course, other solutions to the strong CP problem in this context would be interesting.

One way to decrease the slope after inflation is to tie it to the value of the inflaton σ. We can add the term $\kappa\sigma^2\phi^2$ to the potential. One can check that our parameter space will remain technically natural, essentially because, like the relaxion, the classical value of the inflaton will be large compared to the cutoff. There is now an additional slope, $\kappa\sigma^2 M^2/g$, which we take to be larger than gM^2. Assuming σ has a roughly constant value during most of inflation, we will describe this with a new effective coupling $\tilde{g}^2 = \kappa\sigma^2$ which is constant during most of inflation. The inflation field drops to zero after inflation, removing this new contribution to the potential and leaving the original slope $\sim gM^2$. In order to solve the strong CP problem as well, we need the slope of the potential to drop by a factor of $\theta \lesssim 10^{-10}$ after inflation so that the axion is only displaced by this amount from its (local) minimum. Thus, we require $gM^2 \sim \theta\tilde{g}^2(M^2/g)$, or $g \sim \tilde{g}\sqrt{\theta}$. This has the added benefit that, once the slope drops, every ϕ vacuum that any patch of the Universe sits in now becomes very long-lived because the effective barriers rose by $\gtrsim 10^{10}$. It is easy to show that quantum corrections from this term (assuming $\sigma > M_{\rm pl}$ does not contribute significantly to the ϕ potential).

The condition on the number of e-folds of inflation is now

$$N \gtrsim \theta \frac{H_i^2}{g^2}. \tag{10}$$

The condition equations (5), (6), (7), and (8) become, respectively,

$$H_i > \frac{M^2}{M_{\rm pl}\sqrt{\theta}} \quad \text{(vacuum energy)}, \tag{11}$$

$$H_i < \Lambda_{\rm QCD} \qquad \text{(barriers form)}, \tag{12}$$

$$H_i < \left(\frac{gM^2}{\theta}\right)^{1/3} \quad \text{(classical beats quantum)}, \tag{13}$$

$$gM^2 f \sim \Lambda^4\theta \quad \text{(barrier heights)}. \tag{14}$$

Note that, because of the dropping slope, the vacuum energy Eq. (11) is greater than the fourth power of the cutoff, M^4, by a power of θ^{-1}. This is not a problem for the effective theory, but it may be of concern for the UV completion.

The constraints above give a bound on the cutoff of

$$M < \left(\frac{\Lambda^4 M_{\rm pl}^3}{f}\right)^{1/6}\theta^{1/4} \sim 30\ \text{TeV} \times \left(\frac{10^9\ \text{GeV}}{f}\right)^{1/6}\left(\frac{\theta}{10^{-10}}\right)^{1/4}. \tag{15}$$

This model now satisfies all constraints and has only the QCD axion and inflaton added to the standard model below the cutoff. Thus, the minimal model has no hierarchy problem and no strong CP problem, and it has a natural candidate for dark matter. Because of the constraints, its full parameter space can be probed in a number of ways in future experiments.

Our mechanism for solving strong CP, dropping the slope, potentially allows us to loosen the constraint from requiring classical rolling to dominate [Eq. (13)]. If when the slope drops the sign of the underlying slope is the opposite sign, the small fraction of patches that have not reached the barriers will eventually find themselves with a large negative cosmological constant and should suffer a collapse and not eternal inflation (assuming one of the "typical" patches has our measured cosmological constant). Potentially, we can ignore the classical rolling constraint and allow a higher cutoff—using Eqs. (11) and (12), we derive

$$M < (\Lambda M_{\mathrm{pl}})^{1/2}\theta^{1/4} \sim 1000\ \mathrm{TeV} \times \left(\frac{\theta}{10^{-10}}\right)^{1/4}. \quad (16)$$

One concern about loosening this constraint is that there are a small fraction of patches that naively fluctuate well beyond $\phi \sim M^2/g$. These patches, however, are not a concern if, for example, ϕ is periodic with period $\sim M^2/g$.

A final consistency check is making sure that reheating at the end of inflation does not destabilize ϕ and take us out of a good minimum (one in which the electroweak scale is the correct size). If the standard model fields reheat to temperatures below the temperature where appreciable barriers form (roughly 3 GeV assuming $\theta = 10^{-10}$; see, for example, Ref. [22]), then ϕ remains in its original vacuum, though it can be displaced from its minimum. If the reheat temperature is above this scale, the barriers effectively disappear and the relaxion can begin to roll. We estimate the distance ϕ rolls (and one can show it slow rolls as long as $\Delta\phi < M_{\mathrm{pl}}$) as

$$\frac{\Delta\phi}{f} \sim \frac{\dot{\phi}}{H_b f} \sim \frac{V'}{H_b^2 f} \sim \theta \frac{\Lambda^4}{T_b^4}\frac{M_{\mathrm{pl}}^2}{f^2}, \quad (17)$$

where T_b is the scale where barriers begin to form and H_b is the Hubble scale at this temperature. Taking $T_b \sim 3$ GeV and $\theta = 10^{-10}$, we find that ϕ moves less than one period if $f > 10^{10}$ GeV. However, even for $f = 10^9$ GeV (roughly the lower bound on the QCD axion coupling), while the relaxion rolls through multiple periods, it changes the Higgs squared mass by less than 1 eV^2 for any cutoff above 1 TeV. And because the relaxion still easily satisfies the slow roll condition, it stops rolling once the barriers appear. Thus, the reheat temperature can be larger (as large as the cutoff), without destabilizing the mechanism that chose the small electroweak scale.

Non-QCD model.—Our solution to the hierarchy problem only requires the Higgs VEV to produce barriers which stop ϕ from rolling. If the barriers are produced by something other than QCD, we can avoid the impact on the strong *CP* problem (as it can, for example, be solved by the standard axion), and the barrier heights can be larger than the QCD scale. As we see in the model below, both of these allow for a larger upper bound on the cutoff, though we require a coincidence of scales due to current experimental constraints (similar to the μ problem in the minimal supersymmetric standard model [2]).

The dynamics of this model are similar to the previous one—ϕ rolls until the Higgs VEV is large enough to produce barriers to stop ϕ. The ϕ Lagrangian is the same as in the first model, except that it couples to the $\tilde{G}'^{\mu\nu}G'_{\mu\nu}$ of a new strong group (not QCD), which we take to be $SU(3)$. The Higgs boson couples to new fermions which are charged under both the new strong group and the electroweak group. Its VEV contributes to their masses and raises the barriers when turned on. The upper bound on the cutoff is much larger than the model in section Minimal Model, mostly due to the avoidance of the strong *CP* contributions. The new fermions are required to be at the weak scale, and they are thus collider accessible and impact the Higgs boson and electroweak precision physics.

The new fermions are labeled suggestively as (L, N) and their conjugates (L^c, N^c). The fields L and N carry the same standard model charges as the lepton doublet and the right-handed neutrino, respectively, and are in the fundamental representation of the new strong group, and L^c and N^c are in the conjugate representations. They have Dirac masses and Yukawa couplings with the Higgs boson as follows:

$$\mathcal{L} \supset m_L L L^c + m_N N N^c + y h L N^c + \tilde{y} h^\dagger L^c N. \quad (18)$$

Collider and other constraints require m_L to be greater than the weak scale, but no such constraint exists on m_N, and the barriers in the ϕ potential vanish as the lightest fermion mass goes to zero. Thus, the key is that a Higgs VEV can significantly increase the mass of the lightest fermion at tree level. A naive dimensional analysis estimate of the barrier coefficient (in front of the periodic potential) is $\Lambda^4 \simeq 4\pi f_{\pi'}^3 m_N$, where $f_{\pi'}$ is the chiral symmetry-breaking scale of the new strong group and we have assumed that $m_L \gg f_{\pi'} \gg m_N$.

In this limit, the Higgs VEV gives a contribution to the lightest fermion mass of size $y\tilde{y}\langle h\rangle^2/m_L$. Technical naturalness requires N's Dirac mass to be at least the larger of $\sim(y\tilde{y}/16\pi^2)m_L \log M/m_L$ and $\sim y\tilde{y} f_{\pi'}^2/m_L$, and thus the Higgs VEV only has a significant impact if

$$f_{\pi'} < \langle h\rangle \quad \text{and} \quad m_L < \frac{4\pi\langle h\rangle}{\sqrt{\log M/m_L}}. \quad (19)$$

In addition, for the Higgs boson to have an effect, the lightest fermion, of mass $\sim m_N$, should be lighter than the confinement scale—otherwise, the axion potential will be saturated. The additional constraint is

$$4\pi f_{\pi'} > \frac{y\tilde{y}\langle h\rangle^2}{m_L}. \quad (20)$$

There should be a lower limit on m_L around the weak scale from collider production of L, L^c. In the part of parameter space with the largest allowed $f_{\pi'}$ (and the largest allowed cutoff), the bound should be weaker than that on chargino or neutralino production [23] as only the baryon-like states should leave significant missing transverse energy, while the meson states decay promptly via mixing with the Higgs boson. Another constraint on the Yukawa couplings is from Higgs physics, namely, decays of the Higgs boson to the composite N states. For example, if $y, \tilde{y} \lesssim 0.1$ and $m_L > 250$ GeV, the branching ratio to the new mesons is less than 10%. In addition, there are precision electroweak constraints, which are more important than the Higgs constraints only if m_L is small. Finally, there may be interesting cosmological constraints (or signals) on higher-dimensional operators from the long-lived or stable baryons in this sector. We leave all of these studies for future work.

Thus, the dynamics are exactly those of the model in section Minimal Model, where ϕ rolls, turns on the Higgs VEV, and is stopped by barriers determined by the VEV. The same constraints in Eqs. (5), (7), and (8) apply [in which case Ref. (6) is already satisfied], with $\Lambda^4 \to 4\pi f_{\pi'}^3 m_N \sim 4\pi f_{\pi'}^3 y\tilde{y}\langle h\rangle^2/m_L$. One additional difference is that the ϕ field is no longer the QCD axion, so the bounds on its couplings are much weaker. Assuming f is at least as large as the cutoff, we can parametrize the bound on M as

$$\begin{aligned} M &< (\Lambda^4 M_{\rm pl}^3)^{1/7}\left(\frac{M}{f}\right)^{1/7} \\ &< 3\times 10^8\ \text{GeV}\left(\frac{f_{\pi'}}{30\ \text{GeV}}\right)^{3/7}\left(\frac{y\tilde{y}}{10^{-2}}\right)^{1/7} \\ &\quad \times\left(\frac{300\ \text{GeV}}{m_L}\right)^{1/7}\left(\frac{M}{f}\right)^{1/7} \\ &< 2\times 10^8\ \text{GeV}\left(\frac{f_{\pi'}}{30\ \text{GeV}}\right)^{4/7}\left(\frac{M}{f}\right)^{1/7}, \end{aligned} \tag{21}$$

where, in the last line, we used Eq. (20). In the standard model, a cutoff that saturates this bound would require a tuning of one part in 10^{12}. Here, we have achieved this hierarchy dynamically.

Again, a final constraint comes if reheating occurs above the strong coupling scale. In that case, the relaxion begins to roll and, unlike the QCD case, requiring slow roll is a nontrivial constraint. We require that the energy lost due to Hubble friction is larger then the kinetic energy gained by rolling from one barrier to the next (as they turn on), which amounts to an order one fraction of its kinetic energy

$$(\dot{\phi}H_b)f > \dot{\phi}^2 \sim \Lambda^4, \tag{22}$$

where, again, we take $H_b \sim T_b^2/M_{\rm pl}$, and T_b is the temperature at which the effective barriers appear. We estimate this to be $T_b^4 \sim 16\pi^2 f_{\pi'}^4$ (to match the corresponding temperature in QCD). Using this and our formulas for Λ^4 and the lightest fermion, m_N, we arrive at a lower bound on f:

$$f > (0.05)M_{\rm pl}\left(\frac{y\tilde{y}}{10^{-2}}\right)^{1/2}\left(\frac{30\ \text{GeV}}{f_{\pi'}}\right)^{1/2}\left(\frac{300\ \text{GeV}}{m_L}\right)^{1/2}. \tag{23}$$

One can check to see that this bound also restricts the distance ϕ rolls before the barriers turn on to less than one period: $\Delta\phi < f$.

Note that, from Eq. (19), this model ceases to work properly if either m_L or $f_{\pi'}$ gets much above a few hundred GeV. Thus, in this model, we see that a natural solution to the hierarchy problem requires the existence of new weak-scale electroweak particles charged under a new gauge group which confines below the weak scale. However, these particles need not be charged under QCD, making them harder to detect at hadron colliders. In addition, while precision Higgs boson and electroweak observables depend strongly on the Yukawa couplings, M depends only weakly on them, and thus constraints can be easily evaded without a significant effect on the parameter space.

Example inflation sector.—We need many e-folds of inflation in order to have enough time for the scanning of the Higgs mass. We find it preferable to avoid eternal inflation because then a multiverse is produced which will ultimately populate all of our vacua. Even without eternal inflation, though, most inflation models can easily produce many e-folds. For example, even single-field inflation with an $m^2\sigma^2$ potential (where σ is the inflaton) will produce enough e-folds with the required low Hubble scale when $m \sim 10^{-27}$ GeV. However, it would have to be followed by a second stage of inflation to achieve the observed $\delta\rho/\rho$ and a large enough reheat temperature. It is not surprising that single-field inflation can achieve the required number of e-folds since the constraints on our models are very similar to those on inflation.

In this section we give a simple hybrid inflation model as a proof of principle that achieves all of our requirements on the inflation model and gives the observed $\delta\rho/\rho$. As is a generic issue with many low-scale inflation models, however, this inflation sector is not natural. We will demonstrate a model for the QCD axion solution. The same model works for the non-QCD axion solution and has fewer constraints. In the future we will present a new type of inflation sector based on our mechanism, which is natural and satisfies all of the constraints necessary for our solution to the hierarchy problem [21]. It would be interesting to find other natural models of inflation that also satisfy our constraints.

We consider a hybrid inflation sector [24], with the following relevant terms in the scalar potential:

$$V \ni m^2\sigma^2 + c\sigma^2\chi^2 - m_\chi^2\chi^2 + \lambda\chi^4, \tag{24}$$

where σ is the inflaton and χ is the waterfall field. We must satisfy the constraints on the inflation model in Eqs. (10) and (11). We will take an initial phase of inflation with a super-Planckian field excursion for σ which is followed by a normal hybrid inflation phase driven by the energy in χ. Further, we require $\delta\rho/\rho < 1$ at the beginning of inflation in order to avoid eternal inflation. And observations require $\delta\rho/\rho \approx 10^{-5}$ by the end of inflation. Putting all of these constraints together leaves an open parameter space. One set of parameters which work for the QCD axion model are $M \sim 10^4$ GeV, $f \sim 10^9$ GeV, $\Lambda \sim 10^{-1}$ GeV, $g \sim 10^{-31}$ GeV, $\theta \sim 10^{-10}$, $H_i \sim 10^{-5}$ GeV, final Hubble scale $H_f \sim 10^{-12}$ GeV, and $\lambda \sim 10^{-1}$. Instead of attempting to characterize the entire parameter space, we simply present this one point which works since our goal is just to illustrate that it is possible to find an inflation sector for our model. One could even attempt to make this model natural by supersymmetrizing it, but this model is just meant to demonstrate that the requirements on inflation are potentially satisfiable and, for example, it does not even predict an allowed scalar tilt. (We thank Renata Kallosh for pointing this out.) Thus, significant progress is still to be made with a viable inflation sector.

Observables.—Central to our class of solutions is a new, light, very weakly coupled boson. The most promising ways to detect this field are through low-energy, high-precision experiments. This is in stark contrast to conventional solutions to the hierarchy problem, which require new physics at the weak scale and hence are (at least potentially) observable in colliders. A comprehensive discussion of the experimental program necessary to discover this mechanism is beyond the scope of this work—we will instead highlight experimental strategies that seem promising. While it may be challenging to ultimately confirm our mechanism, it is an open goal which will hopefully motivate new types of searches.

Our class of solutions generically predicts axionlike dark matter. The simplest model predicts the QCD axion as a dark matter candidate. Excitingly, a new area of direct detection experiments focused on light bosons is now emerging [25–44]. These new experiments may, for example, open up the entire QCD axion range to exploration. In the parts of parameter space where the axionlike particle's lifetime is at or below the age of the Universe, there will already be constraints or potential cosmological signals (see, for example, Ref. [45]). It is interesting that light field (axionlike) dark matter candidates in our theories replace the heavy particle (weakly interacting massive particle–like) candidates of conventional solutions to the hierarchy problem.

While our theories can have axion dark matter, the specific prediction for the axion abundance and mass-coupling relation may be altered. Because the axion potential now has an overall slope, it can acquire an initial velocity in the early Universe after reheating set by the slow-roll condition. This would change the calculation of the final axion dark matter abundance and is thus important to work out. We leave this for future work, but we note that this could predict QCD axion dark matter in a region of parameter space that differs from where the standard axion model does. In addition, in the non-QCD case, the field ϕ may be stopped right when the barriers first appear, and therefore the mass of the axion particle may be naturally tuned to be small. This small mass improves the observability of the axion dark matter [26,27]. Interestingly, if the axion is observed, its mass and couplings can be measured and would not satisfy the usual relation with the confinement scale Λ (potentially measurable in colliders). Observation of such dark matter would be a tantalizing hint of our mechanism.

In the QCD case there is a preference for the large θ from Eq. (15). While this is a relatively weak preference because of the $1/4$ power, it does favor a static nucleon electric dipole moment (EDM) that may be observable. (This is in addition to the oscillating EDM induced by the axion in this scenario [25].) Upcoming nucleon EDM experiments are predicted to improve on the current bounds by several orders of magnitude, potentially providing further hints of this scenario (see, for example, Ref. [46]). In the non-QCD case, there can be a large θ in the new strong group. A two-loop diagram may then give EDMs for nucleons or electrons which could be detectable and may even give a constraint on parameter space.

Our models appear to generically require low-scale inflation (unless we find a new dissipation mechanism besides Hubble friction during inflation). This prediction can be falsified by an observation of gravitational waves from inflation, but it cannot be directly observed.

The models presented in this Letter either have a low cutoff (in the QCD case) or new physics at the weak scale (in the non-QCD case). Either case is then potentially observable at the LHC or future colliders. The non-QCD case has new fermions at the weak scale charged under a new strong group with a confinement scale below the weak scale. This scenario should have rich phenomenology; for example, the lightest states are composite singlet scalars that can mix with the Higgs boson. For compositeness scales much smaller than the weak scale, the phenomenology may be similar to Refs. [47–49]. Both direct searches for new fermions with electroweak quantum numbers and more refined measurements of Higgs branching ratios could probe the parameter space of this model, though the latter can be suppressed with small Yukawa couplings without significantly impacting our bounds. Because the non-QCD model fails to be effective without electroweak fermions with masses in the hundreds of GeV, the whole parameter space could conceivably be covered by the LHC and/or a future linear collider. Further studies of optimal strategies are warranted. Observation of this new weak scale physics could provide the first evidence of such a mechanism.

Verification of a critical piece of this class of theories could come by observing the direct coupling of the new light field to the Higgs boson. While this is unlikely to happen in colliders, there may be significant opportunities in new low-energy experiments. As a component of dark matter, oscillations of the new light field cause oscillations of the Higgs VEV. This causes all scales connected to the Higgs boson—for example, the electron mass—to oscillate in time with a frequency equal to the axion mass. Additionally, the new light field couples to matter through its mixing with the Higgs boson and so mediates a new force. It may be possible to design new high-precision experiments to search for these phenomena [50]. Such searches will be quite challenging. However, if axionlike dark matter is discovered first, and thus its mass is measured, that mass can be targeted, greatly enhancing the sensitivity of resonance searches [50].

Discussion.—We have found a new class of solutions to the hierarchy problem. The two models in this Letter are examples of a broader class of theories in which dynamical evolution in the Universe drives the weak scale to its small value. We find that, in order to realize a model of this type, it is necessary to satisfy the following conditions. (i) Dissipation—Dynamical evolution of a field requires energy transfer which must be dissipated in order to allow the field to stop, and hence stop the scan of the Higgs mass. This also allows the model to be insensitive to initial conditions. In our models, dissipation is accomplished by gravity via Hubble friction during inflation. (ii) Self-similarity—Cutoff-dependent quantum corrections will choose an arbitrary point in the scanning field's range at which the Higgs mass is canceled. The scanning field must therefore have a self-similar potential across its entire field range so that the Higgs boson can stop its evolution at any arbitrary point. In our models the periodic axion potential provides this self-similarity. (iii) Higgs backreaction—The Higgs VEV must backreact on the scanning field, stopping the evolution at the appropriate value. In our models this is accomplished in a technically natural way by coupling the Higgs boson to fermions which affect the scanning field's potential. (iv) Long time period—There must be a sufficiently long time period during the early Universe for the Higgs mass to be scanned across the entire range from the cutoff to zero. It would be valuable to find other models in this class [21]. In a sense, this type of theory gives a specific realization of the hope of applying "self-organized criticality" to the weak scale [51].

More can be learned about this class of theories by finding ultraviolet completions. UV completions may impose additional constraints on these models but may also reveal new realizations of this mechanism (e.g., as realized in string theory with axion monodromy or higher-dimensional effective field theory [10–14]).

While we have used this mechanism for the hierarchy problem, it is possible that it could be applied to other naturalness problems. For example, instead of the Higgs boson, it could be used to make other scalar fields light (the inflaton, curvaton, chameleons, etc.). Of course, the biggest naturalness problem is the fine-tuning of the cosmological constant. Perhaps a variant of the mechanism could lead the way to a new solution.

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