



Bose Polarons in the Strongly Interacting Regime

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When an impurity is immersed in a Bose-Einstein condensate, impurity-boson interactions are expected to dress the impurity into a quasiparticle, the Bose polaron. We superimpose an ultracold atomic gas of ^{87}Rb with a much lower density gas of fermionic ^{40}K impurities. Through the use of a Feshbach resonance and radio-frequency spectroscopy, we characterize the energy, spectral width, and lifetime of the resultant polaron on both the attractive and the repulsive branches in the strongly interacting regime. The width of the polaron in the attractive branch is narrow compared to its binding energy, even as the two-body scattering length diverges.

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An impurity interacting with a quantum bath is a simplified yet nontrivial many-body model system with wide relevance to material systems. For example, an electron moving in an ionic crystal lattice is dressed by coupling to phonons and forms a quasiparticle [see Fig. 1(a)] known as a polaron that is an important paradigm in quantum many-body physics [1]. Impurity atoms immersed in a quantum degenerate bosonic or fermionic atomic gas are a convenient experimental realization for Bose or Fermi polaron physics, respectively, where Bose or Fermi refers to the quantum statistics of the bath. The Bose polaron case has been explored in recent theoretical work [2–9] in the weakly interacting regime, where the physics can be described by the Fröhlich model [1]. However, the ability to use a Feshbach resonance [10] to tune the impurity-boson (IB) scattering length a_{IB} opens up the possibility of exploring the Bose polaron in the strongly interacting regime, where physics beyond the Fröhlich model is expected [11–14]. While experiments to date [15–20] have focused on the weakly interacting regime, investigation of the Bose polaron in the strongly interacting regime accesses physics beyond the Fröhlich paradigm, and, moreover, represents a step towards understanding a fully strongly interacting Bose system. While a_{IB} can be tuned to approach infinity, the boson-boson (BB) scattering length a_{BB} can still correspond to the mean-field (MF) limit. An impurity interacting very strongly with a Bose gas that is otherwise in the mean-field regime is, on the one hand, something more difficult to model and to measure than a weakly interacting system. On the other hand it is theoretically more tractable, and empirically more stable, than a single-component “unitary” Bose gas in which a_{BB} diverges and thus every pair of atoms is strongly coupled [21]. The behavior of the Bose polaron in the strongly interacting regime may also affect the efficiency of Feshbach molecule production when ramping across the resonance.

Our experiment employs techniques similar to those used in recent Fermi polaron measurements [22–25].

However, there are important differences between the Bose polaron and the Fermi polaron. From a theory point of view, the Bose polaron problem involves an interacting superfluid environment and also has the possibility of three-body interactions [14], both of which are not present for the Fermi polaron studied thus far. And on the experimental side, both three-body inelastic collisions and the relatively small spatial extent of a BEC (compared to that of the impurity gas) create challenges for measurements of

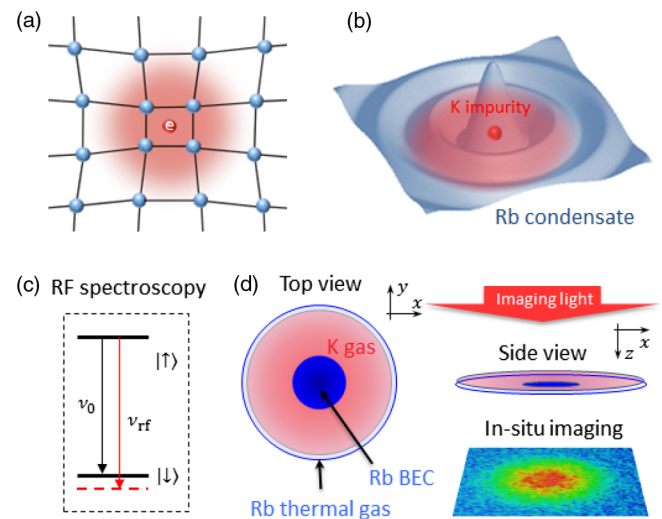


FIG. 1. Impurities immersed in a bosonic bath. (a) Cartoon depictions of the Bose polaron formed by an electron moving in a crystal lattice and (b) its counterpart of an impurity in a continuous system. (c) Radio-frequency (rf) spectroscopy of ^{40}K impurities in a ^{87}Rb Bose-Einstein condensate (BEC). The black lines denote two hyperfine states of bare K atoms and the red dashed line is the shifted energy level due to interactions with the BEC. (d) Geometry of the trapped BEC and impurity clouds. The dark blue represents the Rb BEC cloud, the light blue shows the Rb thermal cloud, and the red shows the K impurity cloud. The imaging light propagates from top to bottom along z .

the Bose polaron. This work, in parallel with work done at Aarhus [26], describes the first experiments performed on Bose polarons in the strongly interacting regime. We report measurements of the Bose polaron energies and lifetimes using rf spectroscopy of fermionic ^{40}K impurities in a BEC of ^{87}Rb atoms. We tune the impurity-boson interactions using a Feshbach resonance, and our measurements reveal both an attractive and a repulsive polaron branch, whose energies agree with recent predictions [11–13]. We find that the Bose polaron exists across the strongly interacting regime and has a larger binding energy than does the low-density, two-body molecular state.

In our experiment, we typically have 2×10^5 Bose-condensed Rb atoms in a harmonic trap whose radial and axial frequencies are $f_{\text{Rb},\rho} = 39$ Hz and $f_{\text{Rb},z} = 183$ Hz, respectively. The Thomas-Fermi radii of the BEC are typically $15 \mu\text{m}$ radially and $3.2 \mu\text{m}$ axially. The peak density of the condensate is $n_{\text{BEC}} = 1.8 \times 10^{14} \text{ cm}^{-3}$. The same optical trap typically contains 2.5×10^4 K atoms, where the trap frequencies are $f_{\text{K},\rho} = 50$ Hz and $f_{\text{K},z} = 281$ Hz. Our experiment usually operates at a temperature of 180 nK, such that the fermionic impurity atoms have $T/T_F \approx 0.4$, where T_F is the Fermi temperature; we expect that their quantum statistics are not important [27]. The peak potassium density is $n_{\text{K}} = 2 \times 10^{12} \text{ cm}^{-3}$. The BEC is weakly interacting with $a_{\text{BB}} = 100 a_0$ [28], where a_0 is the Bohr radius. Following Refs. [11,12], we define momentum and energy scales, respectively, by $k_n = (6\pi^2 \bar{n}_{\text{BEC}})^{1/3}$ and $E_n = \hbar^2 k_n^2 / 2m_{\text{Rb}}$, where \bar{n}_{BEC} is the average probed density and m_{Rb} is the Rb mass. Typically E_n/h and k_n are 25 kHz and $1/(900 a_0)$, respectively. The typical thermal energy is 15% of E_n , and thus we expect that the finite momentum of the impurities does not play a significant role. We tune the impurity-boson interactions using a broad s -wave Feshbach resonance for potassium atoms in the state $|\downarrow\rangle \equiv |f = 9/2, m_f = -9/2\rangle$, and ^{87}Rb atoms in the $|1, 1\rangle$ state [29]. The impurity-boson scattering length a_{IB} as a function of magnetic field B is given by $a_{\text{IB}} = a_{bg}[1 - \delta B / (B - B_0)]$, where $a_{bg} = -187 a_0$, $B_0 = 546.62$ G, and $\delta B = -3.04$ G [30].

We measure the energy spectrum by performing rf spectroscopy. To minimize the loss due to three-body inelastic collisions, we initially prepare the impurity atoms in a state, $|\uparrow\rangle \equiv |9/2, -7/2\rangle$, that is weakly interacting with the bosons. We then apply a rf pulse at a frequency ν_{rf} , detuned from the bare atomic transition ν_0 , to drive impurity atoms into the strongly interacting $|\downarrow\rangle$ state [see Fig. 1(c)]. Immediately after the rf pulse, we selectively image the impurity atoms in the $|\downarrow\rangle$ state. The rf photon has frequency ν_{rf} over a range of 79.9 to 80.5 MHz. The rf pulse has a Gaussian envelope with the rf power proportional to $\exp(-t^2/2\Delta t^2)$. The Gaussian envelope is truncated at $\pm 4\Delta t$ and the full pulse duration is $8\Delta t$. The rf line shape is approximately proportional to $\exp(-\nu_{\text{rf}}^2/2\delta\nu^2)$, where the frequency width is $\delta\nu = 1/(4\pi\Delta t)$. In the experiment, we

used long π pulses of width $\Delta t = 65 \mu\text{s}$ ($\delta\nu = 1.2 \text{ kHz} = 0.05E_n/h$) for most of the spectroscopy data. In order to improve signal for the spectra around the unitarity regime, we used short 1.5π pulses of width $\Delta t = 12.5 \mu\text{s}$ ($\delta\nu = 5.7 \text{ kHz} = 0.23E_n/h$).

In Figs. 2(a)–2(c), we show typical rf spectroscopy data at a relatively weak interaction strength. The total number of spin-flipped atoms [Fig. 2(a)] as a function of ν_{rf} is dominated by a signal from ^{40}K atoms that do not spatially overlap with the BEC [see Fig. 1(d)]. To distinguish a signal that comes from ^{40}K atoms that overlap with the BEC, we apply a cut based on the root-mean-square (rms) width obtained by fitting the image to a two-dimensional Gaussian distribution [Fig. 2(b)]. For those imaged clouds whose size is less than or equal to $11 \mu\text{m}$ (red points), we apply an inverse Abel transform to extract the central density [31]. The normalized central density, averaged over a region corresponding to a radius of $R_{\text{avg}} = 5 \mu\text{m}$ in the transverse direction, is shown in Fig. 2(c). The measured density is normalized by a calculated initial central density of the impurity atoms [31]. We use a Gaussian fit [red line in Fig. 2(c)] to this spectrum to obtain the energy shift, Δ , defined as the fit peak (corrected for initial interactions), and the spectral width. Figure 2(d) shows rf spectra similar to Fig. 2(c) but at different interaction strengths. The widths of almost all of the spectra remain sufficiently small that a clear peak is observed, which is consistent with a quasi-particle description of the excitation.

Figure 3 shows the fit energy shifts and spectral widths. To extract the polaron energy from the rf spectra, we need to account for the initial mean-field shift. The energy shift Δ is given by

$$\Delta = h(\nu_0 - \nu_p) + E_{bg}, \quad (1)$$

where ν_p is the fit peak; the background shift due to weak interactions between the initial $|\uparrow\rangle$ state and the Rb BEC is given by $E_{bg} = g_{bg}\bar{n}_{\text{BEC}}$, where $g_{bg} = 2\pi a_{bg}\hbar^2/\mu_{\text{KRb}}$. For comparison, we show the predicted energy of the Bose polaron at zero temperature and momentum for the attractive and repulsive branches as calculated in the T -matrix approach of Ref. [11]. Similar results have been obtained in Ref. [12]. For weak interactions away from the Feshbach resonance (large $|1/k_n a_{\text{IB}}|$), the theoretical energies are well described by a mean-field shift given by $\Delta_{\text{MF}}/E_n = (2m_{\text{Rb}}/3\pi\mu_{\text{KRb}})(k_n a_{\text{IB}})$ (dashed lines in Fig. 3), where μ_{KRb} is the reduced mass between potassium and rubidium atoms. On the positive- a_{IB} side of the resonance, the attractive branch is predicted to asymptotically approach the energy of the Feshbach molecule. Also shown is the universal density-independent prediction for the molecule energy, $E_{\text{mol}}/E_n = -(2m_{\text{Rb}}/\mu_{\text{KRb}})(1/k_n a_{\text{IB}})^2$, and measurements of the binding energy of weakly bound molecules. These latter data are separately measured by rf association of K atoms immersed in a very low-density

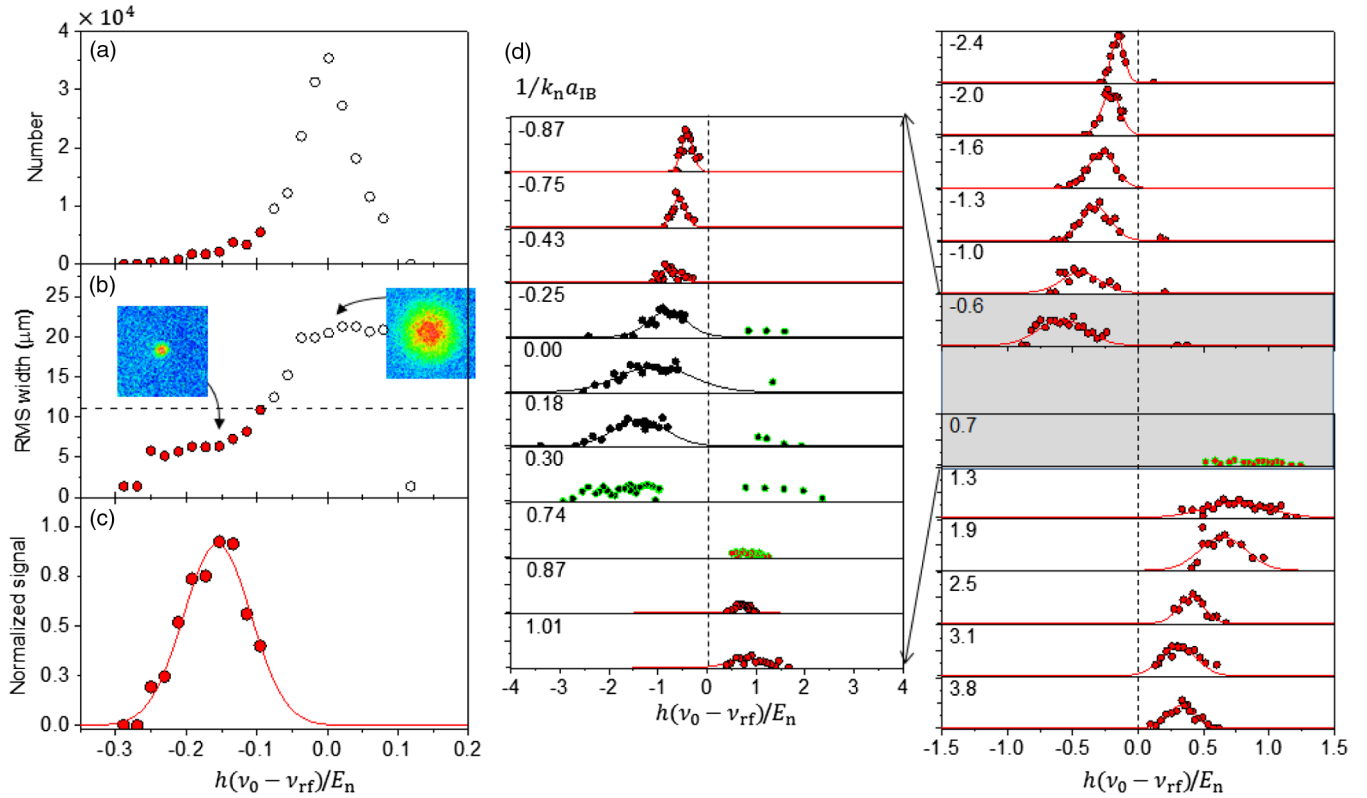


FIG. 2. Radio frequency spectra of ^{40}K impurities in a ^{87}Rb BEC. At the relatively weak interaction of $1/k_n a_{IB} = -2.4$, a typical rf spectrum of impurities is shown in (a)–(c). (a) The total number of atoms spin flipped into $|\downarrow\rangle$ impurity atoms with variable rf frequencies. (b) The rms width of the imaged impurity cloud. The filled points correspond to data where the width is less than $11 \mu\text{m}$ (dashed line). The insets show absorption images at two values of ν_{rf} . (c) The normalized central density of the impurity atoms, obtained using an inverse Abel transform. The red line shows a fit to a Gaussian function. (d) Radio frequency spectra, as shown in (c), for different interaction strengths. Red and black dots show data taken with long and short rf pulses (see text), respectively. Data shown in green circles were excluded from the Gaussian fits.

noncondensed Rb gas, and are similar to measurements reported in Refs. [30,32].

We find that the measured energy shifts match well with the predictions for the Bose polaron across the entire range of impurity-boson interactions. In particular, our data are consistent with an attractive polaron (red line) that exists across unitarity ($1/k_n a_{IB} = 0$) and has an energy that asymptotically approaches that of the low-density molecular state (gray line). When trying to extend the range of the data in the strongly interacting regime, we observe a signal in the measured rf spectrum, but no clear peak [see, for example, the green points in Fig. 2(d)].

The theory, which does not include three-body interactions, predicts a finite lifetime for the repulsive branch. This results in spectral broadening as indicated by the blue shaded region in Fig. 3. In contrast, the attractive polaron is modeled as the ground state of the system, and as such, should exhibit no lifetime broadening. More generally, we do expect three-body loss to more deeply bound states to occur at all interaction strengths, including on the attractive branch. To measure this decay, we spin flip the impurities into the polaron state and hold for a variable amount of

time before imaging. We choose a value of ν_{rf} close to the measured polaron peak and measure the central density of $|\downarrow\rangle$ atoms as a function of the hold time. Figure 4(a) shows example lifetime measurements for the attractive polaron at three different interaction strengths. We obtain a decay rate Γ by fitting to an exponential and the results are presented in Fig. 4(b).

For the attractive branch (data for $1/k_n a_{IB} < 1$), the decay rate increases dramatically as the interaction strength is increased and continues to increase across $1/k_n a_{IB} = 0$. For comparison, the solid line in Fig. 4(b) shows the expected three-body recombination decay rate in the weakly interacting regime ($|1/k_n a_{IB}| > 1$), which scales as $(k_n a_{IB})^4$ [34] and has a coefficient from previous measurements [33]. There is currently no theory for three-body loss in an ultracold gas in the strongly interacting regime. The measured upper limit for the polaron decay rate has a similar magnitude and trend as the expected loss rate for three-body recombination, which suggests that three-body inelastic processes involving an impurity atom and two bosons play an important role in the decay rate measurements. For the repulsive branch, we measure a

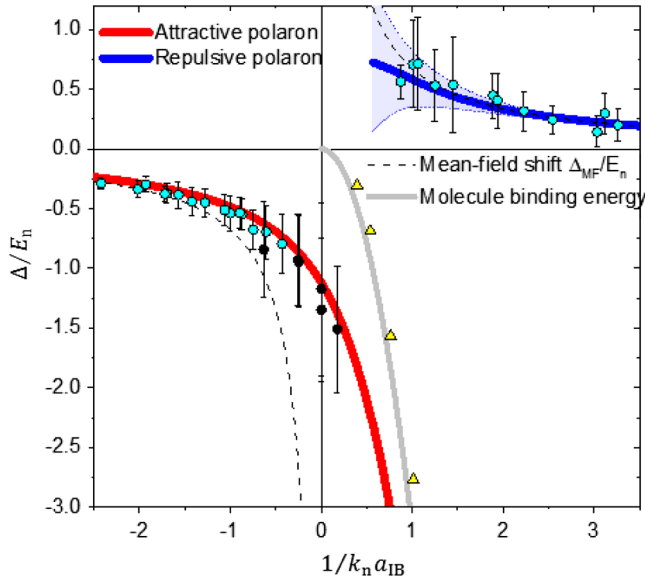


FIG. 3. Energy shift, Δ , and spectral width of Bose polarons. Cyan and black dots show data taken with long and short rf pulses, respectively. Error bars indicate the measured rms spectral width. The mean-field prediction is shown with dashed lines. Recent $T = 0$ Bose polaron energy predictions [11,12] for the attractive and repulsive branches are shown with red and blue lines, respectively. The gray line shows the universal two-body prediction for the energy of KRb Feshbach molecules. Triangles show separate measurements of this energy using rf association of molecules performed in a very low-density Rb gas. This two-body result is not valid for the high-density regime and is shown only for reference. The blue shaded area indicates the predicted spectral width of the repulsive branch.

decay rate that is independent of the interaction strength. We attribute this to dynamics that result from the fact that, unlike on the attractive branch, the repulsive polarons are not confined by the BEC. The dashed line shows a rough estimate of the time that it takes a K atom to leave the central region that we probe.

The measured loss rate, along with the rf pulse linewidth, contribute to the total width of the rf spectra indicated by the error bars in Fig. 3. However, our measured widths are generally larger than the quadrature sum of these broadening effects. On the attractive (repulsive) branch, the spectra are sometimes broader by as much as a factor of 3 (8). Measured widths increase for stronger interaction strengths (smaller $|1/k_n a_{IB}|$) and are larger for the repulsive branch than for the attractive branch. While the spectral width contains information about the polaron lifetime, there are several experimental factors that can also contribute to broadening. We estimate that density inhomogeneity introduces broadening by up to 15% of the shift at the center of the polaron peak, Δ . Beyond this, the rf spectra could additionally be broadened by intrinsic many-body effects [26,35], the thermal momentum distribution of impurities, or the fact that the data are not taken in the linear response regime. It must be acknowledged, however, that if this

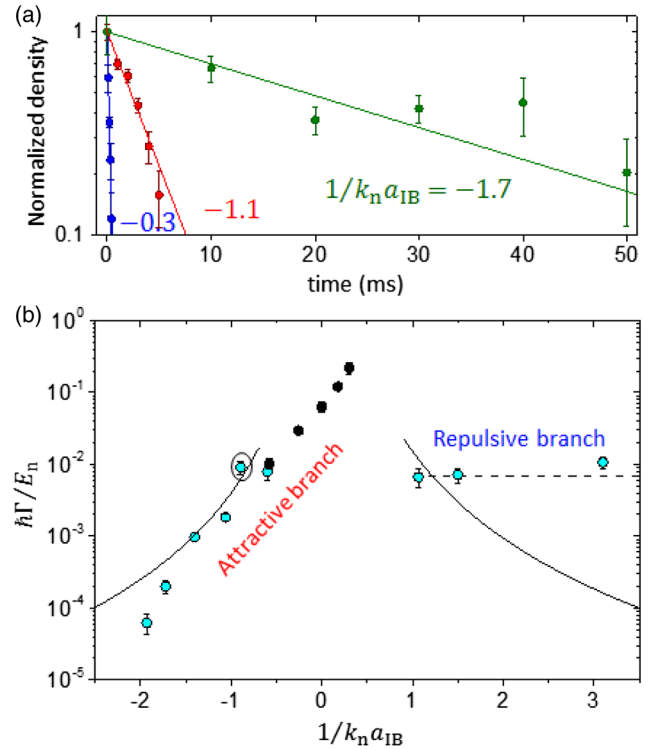


FIG. 4. Decay rate of the Bose polaron. (a) Example decay rate measurements. Each point is the average of five shots with the error bar being the standard error of the mean. Lines are fits to an exponential with a decay rate, Γ . (b) Decay rates at different interaction strengths. Cyan and black dots denote the same rf pulses as in Fig. 3. The solid lines are the predicted three-body recombination loss rate. The dashed line shows the estimated rate for atoms to leave the probed central region of the BEC: $1/(R_{\text{avg}}/\sqrt{k_B T/m_K})$, where $R_{\text{avg}} = 5 \mu\text{m}$. The circled dot is near a narrow d -wave Feshbach resonance [33], which could cause an increased decay rate.

unaccounted-for width gives rise to asymmetric line shapes, there will be systematic errors in our energy determination. The excess broadening, in any case, gives a lower limit on the polaron lifetime, which is still longer than the quasiparticle's inverse energy.

This first measurement of the Bose polaron in a three-dimensional trapped atom gas probed the energies and lifetimes for both the attractive and repulsive branches. A future direction would be to probe other basic quasiparticle properties, such as the quasiparticle's effective mass and residue. In addition, it would be interesting to explore the dependence of the excitation spectrum on temperature, both theoretically and experimentally. In our case, by probing impurities in the central region of the BEC, the effective temperature of the bosons should be very close to 0. Finally, Ref. [14] theoretically considers the effect of Efimov trimers on the Bose polaron. However, for our system, the location of the Efimov resonance is predicted to be $a_- < -30,000 a_0$ [36], such that Efimov effects on the Bose polaron would only occur in an extremely narrow

regime of small $|1/k_n a_{\text{IB}}|$ that we do not resolve. Experimental investigation of Efimov effects on the Bose polaron thus requires a different atomic system with a more favorable three-body interaction parameter.

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Note added.—Recently, we learned of interesting parallel experimental work [26].

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