ဖွာ

Experimental Rectification of Entropy Production by Maxwell's Demon in a Quantum System

Patrice A. Camati,¹ John P. S. Peterson,² Tiago B. Batalhão,¹ Kaonan Micadei,¹ Alexandre M. Souza,²

Roberto S. Sarthour,² Ivan S. Oliveira,² and Roberto M. Serra^{1,3,*}

²Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro, Rio de Janeiro, Brazil

³Department of Physics, University of York, York YO10 5DD, United Kingdom

(Received 26 May 2016; published 5 December 2016)

Maxwell's demon explores the role of information in physical processes. Employing information about microscopic degrees of freedom, this "intelligent observer" is capable of compensating entropy production (or extracting work), apparently challenging the second law of thermodynamics. In a modern standpoint, it is regarded as a feedback control mechanism and the limits of thermodynamics are recast incorporating information-to-energy conversion. We derive a trade-off relation between information-theoretic quantities empowering the design of an efficient Maxwell's demon in a quantum system. The demon is experimentally implemented as a spin-1/2 quantum memory that acquires information, and employs it to control the dynamics of another spin-1/2 system, through a natural interaction. Noise and imperfections in this protocol are investigated by the assessment of its effectiveness. This realization provides experimental evidence that the irreversibility in a nonequilibrium dynamics can be mitigated by assessing microscopic information and applying a feed-forward strategy at the quantum scale.

DOI: 10.1103/PhysRevLett.117.240502

Connections between thermodynamics and information theory have been producing important insights and useful applications in the past few years, which has turned out to be a dynamic field [1-4]. Its genesis traces back to the famous Maxwell's demon gedanken experiment [5-9]. In 1867, Maxwell conceived a "neat fingered being," which has the ability to gather information about the microscopic state of a gas and use this information to transfer fast particles to a hot medium and slow particles to a cold one, engendering an apparent conflict with the second law of thermodynamics. Several approaches and developments concerning this conundrum had been put forward [5–9], but only after more than a century, in 1982, Bennett [10] realized that the apparent contradiction with the second law could be puzzled out by considering the Landauer's erasure principle [11–14].

Theoretical endeavors to incorporate information into thermodynamics acquire a pragmatic applicability within the recent technological progress, where information just started to be manipulated at the micro- and nanoscale. A modern framework for these endeavors has been provided by explicitly taking into account the change, introduced in the statistical description of the system, due to the assessment of its microscopic information [15]. This outlines an illuminating paradigm for Maxwell's demon, where the information-to-energy conversion is governed by fluctuation theorems, which hold for small systems arbitrarily far from equilibrium [16–20]. Generalizations of the second law in the presence of feedback control can be obtained from this framework, establishing bounds for information-based work extraction [21]. Notwithstanding its fundamental relevance, these relations do not provide a clear recipe for building a demon in a laboratory setting. Owing to the challenges associated with a high precision microscopic control, there are only a handful of very recent experiments addressing the information-to-energy conversion at small scales, using Brownian particles [22,23], single electrons [24–26], and laser pulses [27] regarding the classical scenario, where quantum coherence effects are absent. In the quantum context, there are only two experimental attempts related with information-to-energy conversion. The heat dissipated during a global systemreservoir unitary interaction was investigated in a spin system [28] and single photons in nonthermal states were employed to build a thermodynamics-inspired separability criterion [29].

Here, we contribute to the aforementioned efforts deriving an equality concerning the information-to-energy conversion for a quantum nonunitary feedback process. Such relation involves a trade-off between informationtheoretic quantities that provides a recipe to design and implement an efficient Maxwell's demon in a quantum system where coherence is present. Supported by this tradeoff relation and employing Nuclear Magnetic Resonance (NMR) spectroscopy [30–32], we set up an experimental coherent implementation of a measurement-based feedback. Furthermore, we quantify experimentally the effectiveness of this Maxwell's demon to rectify entropy

¹Centro de Ciências Naturais e Humanas, Universidade Federal do ABC,

Avenida dos Estados 5001, 09210-580 Santo André, São Paulo, Brazil



FIG. 1. Illustration of a Maxwell's demon operation. The system starts in the equilibrium state ρ_0^{eq} and it is unitarily driven (\mathcal{U}) to a nonequilibrium state. Then the demon makes a projective measurement, \mathcal{M} , yielding the outcome l with probability p(l). The feedback operation $\mathcal{F}^{(k)}$ is applied with error probability p(k|l). The environment temperature is kept fixed and the whole operation is much faster than the system decoherence time.

production, due to quantum fluctuations [33,34], in a nonequilibrium dynamics.

Theoretical description.-Consider the scenario illustrated in Fig. 1. The working system is a small quantum system, initially in the equilibrium state ρ_0^{eq} (at inverse temperature $\beta = (k_B T)^{-1}$, with k_B being the Boltzmann constant). Later on Maxwell's demon will also be materialized through a microscopic quantum memory. Suppose that the working system is driven away from equilibrium by a fast unitary time-dependent process, \mathcal{U} , up to time τ_1 (driving the system Hamiltonian from \mathcal{H}_0 to \mathcal{H}_{τ_1}). The purpose of the control mechanism is to rectify the quantum fluctuations introduced by this nonequilibrium dynamics. To this end, the demon acquires information about the system's state through a complete projective measurement, $\{\mathcal{M}_l\}$, yielding the outcome l with probability p(l) =tr[$\mathcal{M}_{l}\mathcal{U}\rho_{0}^{eq}\mathcal{U}^{\dagger}$]. Based on the outcome of this measurement, a controlled evolution will be applied. It will be described by unital quantum operations $\mathcal{F}^{(k)}$ [$\mathcal{F}^{(k)}(1) = 1$ for every k], which may include a drive on the system's Hamiltonian from \mathcal{H}_{τ_1} to $\mathcal{H}_{\tau_2}^{(k)}$, along the time interval $\tau_2 - \tau_1$ [35]. Furthermore, we consider the possibility of error in the control mechanism, assuming a conditional probability p(k|l) of implementing the feedback process k (associated with the outcome k) when l is the actual observed measurement outcome. By a suitable choice of the operations $\{\mathcal{F}^{(k)}\}\$, the feedback control mechanism can balance out the entropy production due to the nonequilibrium drive \mathcal{U} . A similar protocol might also be employed in information-based work extraction protocols.

Following the scenario presented above, an integral fluctuation relation can be derived [36,38,39] as

$$\left\langle e^{-\beta(W-\Delta F^{(k)})-I^{(k,l)}}\right\rangle = 1,\tag{1}$$

where W is the stochastic work done on the system, $\Delta F^{(k)} = -\beta^{-1} \ln Z_{\tau_2}^{(k)} / Z_0 \quad [\text{with } Z_t^{(k)} = \text{tr}(e^{-\beta \mathcal{H}_t^{(k)}}) \quad \text{and}$ $Z_0 = \operatorname{tr}(e^{-\beta \mathcal{H}_0})]$, is the free energy variation for the *k*th feedback process, $I^{(k,l)} = \ln p(k|l)/p(k)$ is the unaveraged mutual information between the working system and the control mechanism employed $[p(k) = \sum_l p(k|l)p(l)]$ is the marginal probability distribution of the controlled operation]. The average is computed according to a work distribution probability P(W) that depends on both the measurement and the feedback processes. Equation (1) has the same structure of Sagawa and Ueda's classical relation [40,41]. It is also the generalization of the Tasaki quantum identity obtained for unitary control [42], which was previously discussed in Refs. [38,39]. Jensen's inequality for convex functions can be used to obtain a lower bound for the mean nonequilibrium entropy production

$$\langle \Sigma \rangle \equiv \beta \langle W - \Delta F^{(k)} \rangle \ge - \langle I^{(k,l)} \rangle.$$
⁽²⁾

If the feedback control is absent, Eq. (2) reduces to the standard Clausius inequality, $\langle \Sigma \rangle \geq 0$. On the other hand, Eq. (2) generalizes the second law, elucidating that the correlations between the system and the demon, expressed by the mutual information $\langle I^{(k,l)} \rangle$, may be employed to decrease the entropy production beyond the conventional thermodynamic limit. Besides its material importance to the understanding of the underneath gear of Maxwell's demon, Eq. (2) does not shed light on how to design an efficient feedback-control protocol. Notice that the right-hand side (rhs) of Eq. (2) is unrelated to the specific form of the feedback operations $\{\mathcal{F}^{(k)}\}$, it is only associated with the feedback error probability p(k|l) and the marginal distribution p(k). Therefore, performance analysis of different types of feedback operations is beyond the scope of the bound in Eq. (2).

We bridge such a gap by deriving an equality for entropy production in the presence of feedback control with experimental relevance for the effective design of Maxwell's demon, expressed as [36]

$$\langle \Sigma \rangle = -\mathcal{I}_{gain} + \langle S_{KL}(\rho_{\tau_2}^{(k,l)} || \rho_{\tau_2}^{(k,eq)}) \rangle + \langle \Delta S^{(k,l)} \rangle_{\mathcal{F}}, \quad (3)$$

with only information-theoretic quantities on the rhs. The information gain $\mathcal{I}_{gain} = S(\rho_{\tau_1}) - \sum_l p(l)S(\rho_{\tau_1}^{(l)})$ quantifies the average information that the demon obtains reading the outcomes of the measurement \mathcal{M} [43–49], with $\rho_{\tau_1} = \mathcal{U}\rho_0^{eq}\mathcal{U}^{\dagger}$ being the system's state before the measurement; $\rho_{\tau_1}^{(l)}$ the *l*th postmeasurement state which occurs with probability p(l), and $S(\rho)$ the von Neumann entropy. It is always non-negative for projective measurements [43–46] and it can be interpreted as the reciprocal to the quantity of disturbance impinged on the quantum system due to the measurement operation [47] (see also Ref. [36]). The Kullback-Leibler (KL) relative entropy, $S_{\rm KL}(\rho_{\tau_2}^{(k,l)}||\rho_{\tau_2}^{(k,eq)}) = {\rm tr}[\rho_{\tau_2}^{(k,l)}(\ln\rho_{\tau_2}^{(k,l)} - \ln\rho_{\tau_2}^{(k,eq)})]$, expresses the information divergence between the resulting state of

the feedback-controlled process, $\rho_{\tau_2}^{(k,l)}$, and the equilibrium state for the final Hamiltonian $\mathcal{H}_{\tau_2}^{(k)}$ in the *k*th feedback process, $\rho_{\tau_2}^{(k,eq)} = e^{-\beta \mathcal{H}_{\tau_2}^{(k)}} / Z_{\tau_2}^{(k)}$. The last term, $\langle \Delta S^{(k,l)} \rangle_{\mathcal{F}} = \langle S(\rho_{\tau_2}^{(k,l)}) - S(\rho_{\tau_1}^{(l)}) \rangle_{\mathcal{F}}$, is the averaged change in von Neumann entropy due to the quantum operation $\mathcal{F}^{(k)}$.

The nonequilibrium entropy production in Eq. (3) is negative if and only if

$$\mathcal{I}_{gain} > \langle S_{KL}(\rho_{\tau_2}^{(k,l)} || \rho_{\tau_2}^{(k,eq)}) \rangle + \langle \Delta S^{(k,l)} \rangle_{\mathcal{F}}.$$
 (4)

This provides a necessary and sufficient condition to implement an effective Maxwell's demon for the nonunitary protocol considered here. Equation (3) also encompasses the bound $\langle \Sigma \rangle \geq -\mathcal{I}_{gain}$, which is similar to the bounds previously obtained in Refs. [50,51] considering a different context. In the literature concerning the thermodynamics of information, feedback processes are often regarded as unitary. In this case, the last term of the rhs of Eq. (3) does not contribute. Since the postmeasurement state $\rho_{\tau_1}^{(l)}$ is pure, the average KL relative entropy, $\langle S_{\text{KL}}(\rho_{\tau_2}^{(k,l)}||\rho_{\tau_2}^{(k,\text{eq})})\rangle$, will never be zero for a unitary feedback implemented upon projective measurements (at finite temperature). In a different manner, a nonunitary feedback process can be designed to cancel the term $\langle S_{\text{KL}}(\rho_{\tau_2}^{(k,l)}||\rho_{\tau_2}^{(k,eq)})\rangle$, but in this case the variation of the von Neumann entropy $\langle \Delta S^{(k,l)} \rangle_{\mathcal{F}}$, due to a nonunitary operation, is not null. Along these lines, the trade-off concerning these quantities in Eqs. (3) and (4) empowers the effective design of Maxwell's demon through the performance assessment of different strategies for the controlled operations $\mathcal{F}^{(k)}$.

Experimental implementation.—We employed ¹³C-labeled CHCl₃ liquid sample and a 500 MHz Varian NMR spectrometer to implement and characterize the aforementioned entropy rectification protocol. The spin 1/2 of the ¹³C nucleus is the working system whereas the ¹H nuclear spin plays the role of a quantum memory for Maxwell's demon. Chlorine isotopes' nuclei can be disregarded providing only mild environmental effects due to the fast relaxation of its energy levels. Details on the experimental setup are provided in Ref. [36]. Using spatial average techniques the joint initial state, equivalent to $|0\rangle_{\rm H} \langle 0|\rho_0^{\rm eq,C}$, is prepared, where the $^{13}{\rm C}$ is in an equilibrium state of the initial Hamiltonian, $\mathcal{H}_0^{\rm C} = \frac{1}{2} \hbar \omega_0 \sigma_z^{\rm C}$ (with $\omega_0/2\pi = 2$ kHz, $\sigma_{x,y,z}$ being the Pauli matrices, $|0\rangle_{\rm H,C}$ and $|1\rangle_{\rm H,C}$ representing the excited and ground state of $\sigma_{\tau}^{\text{H,C}}$, respectively). We consider an initial driving protocol as a sudden quench process, described by a quick change in the carbon Hamiltonian from $\mathcal{H}_0^{\rm C}$ to $\mathcal{H}_{\tau_1}^{\rm C} = \frac{1}{2}\hbar\omega_1\sigma_x^{\rm C}$ (with $\omega_1/2\pi = 3$ kHz). The idea is to change the Hamiltonian so quickly that the state of the system remains unchanged. This state will suddenly become far from





FIG. 2. Protocol for the measurement-based feedback. (a) Sketch of the implemented quantum circuit. (b) Choi-Jamiolkowski matrix, χ (with elements $\chi_{s,l}$), of the experimental quantum processes tomography for the nonselective projective measurement on the ¹³C nuclear spin, represented by the map, $\mathcal{M}(\rho) = \sum_{s,l=1,x,y,z} \chi_{s,l} \Xi_s^C \rho \Xi_l^{C\dagger}$ ($\Xi_{\alpha}^C = \mathbb{1}, i\sigma_x^C, i\sigma_y^C, i\sigma_z^C$). The ideal process, described by a no-selective measurement on the $\mathcal{H}_{\tau_1}^{C}$ energy basis, is $\mathcal{M}^{id}(\rho) = \frac{1}{2}(\rho + \sigma_x^{C}\rho\sigma_x^{C})$. For this operator representation choice, a unital process is described by a real process matrix. The imaginary part of the experimental elements $\chi_{s,l}$ are neglectful. (c) The demon effectiveness is quantified by the process trace distance $\delta = \frac{1}{2} tr |\chi^{exp} - \chi^{id}|$ between the experimentally implemented map, χ^{exp} (for the whole protocol: measurement and feedback control) and the map describing the ideal protocol, χ^{id} , as a function of the control mismatch (p(0|1)). The residual error for the zero mismatch $(\delta \approx 4.5\%)$ is due to nonidealities in the protocol implementation. See main text and Ref. [36] for details.

equilibrium, even including coherence in the energy basis of $\mathcal{H}_{\tau_1}^{C}$. The quantum fluctuations, work distribution, and the entropy production in this highly nonadiabatic transformation can be experimentally characterized, in an NMR setting, according to the approach presented in Refs. [33,34]. In the present experiment, this sudden quench is implemented effectively by a short transversal radio-frequency (rf) pulse resonant with the ¹³C nuclear spin (with time duration about 9 μ s) represented by the operation \mathcal{U} [as in Fig. 2(a)].

The feedback mechanism employed is sketched in Fig. 2(a), where the whole feedback operation is much faster than the typical decoherence times, which are on the order of seconds [36]. After the sudden quench (\mathcal{U}), information is acquired by the demon via the natural J coupling between ¹³C and ¹H nuclei, $\frac{1}{2}\pi J\hbar\sigma_z^H\sigma_z^C$ (with J = 215.15 Hz), under a free evolution lasting for about 6.97 ms (equivalent to a CNOT gate). An effective non-selective projective measurement in the energy basis of $\mathcal{H}_{\tau_1}^C$



FIG. 3. Experimental entropy rectification and information quantities. (a) Mean nonequilibrium entropy production $(\langle \Sigma \rangle)$ in the measurement-based feedback protocol as a function of the initial temperature $(k_B T)$ and the control basis mismatch (p(0|1)). The negative values are associated with entropy rectification by Maxwell's demon. (b),(c) Entropy production (black), information gain bound, $\langle \Sigma \rangle \ge -\mathcal{I}_{gain}$ (red), and mutual information bound $\langle \Sigma \rangle \ge -\langle I^{(k,l)} \rangle$ (dark yellow). (d),(e) Measured information quantities appearing in the trade-off relation Eq. (3), i.e., information gain (red), von Neumann entropy variation (blue), and KL relative entropy (green).

is accomplished with an additional longitudinal field gradient, ξ_1 (applied during 3 ms). It introduces a full dephasing on the *z* component of the memory state. This free evolution followed by dephasing correlates the state of the working system (¹³C) with the demon's memory (¹H) leading to a joint "postmeasurement" state equivalent to $|0\rangle_{\rm H} \langle 0|\mathcal{M}_0\rho_0^{\rm C}\mathcal{M}_0 + |1\rangle_{\rm H} \langle 1|\mathcal{M}_1\rho_0^{\rm C}\mathcal{M}_1$, where \mathcal{M}_0 and \mathcal{M}_1 are the eigenbasis projectors for $\mathcal{H}_{\tau_1}^{\rm C}$ with experimentally probed outcome probabilities, $p(l) = 50.0 \pm 0.4\%$ for l = 0, 1, as expected for the sudden quench implemented.

Quantum process tomography (QPT) [52] is applied to verify how effective is the demon's nonselective measurement, with the results displayed Fig. 2(b). The experimentally implemented measurement is very close to the ideal one. In order to investigate the robustness of the feedback process against a control mismatch, we also introduce, in the protocol of Fig. 2(a), a rotation $\mathcal{R}_{\varphi}^{H}$ along the *x* direction on the ¹H spin, in such a way that the feedback error probability, $p(0|1) = p(1|0) = \sin^2(\varphi/2)$, is changed varying the mismatch angle φ . Figure 2(c) displays the trace distance between the experimental and ideal quantum processes for the demon operation as a function of such an error.

Entropy rectification is achieved by a controlled evolution of the ¹³C nuclear spin guided by the demon's memory (encoded in the ¹H nuclear spin state). Such conditional evolution is implemented by the operations Υ_1 and Υ_2 represented in Fig. 2(a) produce ideally the controlled transformation $\Upsilon_2\Upsilon_1 = |\phi_0\rangle_H \langle \phi_0| \mathcal{V}_{\tau}^{(0)} + |\phi_1\rangle_H \langle \phi_1| \mathcal{V}_{\tau}^{(1)}$, where the mismatched control basis are given by $|\phi_0\rangle_H = \cos{(\varphi/2)}|0\rangle_H - i\sin{(\varphi/2)}|1\rangle_H$ and $|\phi_1\rangle_H$ its orthogonal complement; $\mathcal{V}_{\tau}^{(0)} = e^{-i\pi\sigma_y^{C}/4}e^{-i\gamma\sigma_x^{C}/2}$ and $\mathcal{V}_{\tau}^{(1)} = \mathcal{V}_{\tau}^{(0)}\sigma_x^{C}$ are the feedback operations applied on the carbon nucleus, with $\gamma = 2 \arccos (1 - e^{-\beta\hbar\omega_1})^{-1/2}$. Both controlled operations are put into action by a free evolution under the natural *J* coupling $(\frac{1}{2}\pi J\hbar\sigma_z^{H}\sigma_z^{C})$ combined with individual rotations driven by rf-fields resonant with both Larmor frequencies of ¹³C and ¹H nuclei. We have chosen feedback operations where the system Hamiltonian is not driven, in this case $\mathcal{H}_{\tau_2}^{(0)C} = \mathcal{H}_{\tau_2}^{(1)C} = \mathcal{H}_{\tau_2}^{C} = \mathcal{H}_{\tau_1}^{C}$. The concluding step for implementing the controlled operations $\{\mathcal{F}^{(k)}\}$, is a full dephasing in the eigenbasis of $\mathcal{H}_{\tau_2}^{C}$. It is supplied by a second longitudinal field gradient, ξ_2 , and local rotations of the carbon nuclear spin in order to set the dephasing basis.

Performing quantum state tomography (QST) [31] along the experimental implementation of the demon protocol, we can obtain all the information-theoretic quantities in rhs of Eq. (3) (for details see Fig. S2 and the Data Acquisition section in [36]). Figure 3(a) displays the entropy production in the feedback controlled operation implemented in our experiment. We achieved negative values showing the realization of entropy rectification, whose effectiveness worsens as the bases mismatch increases. In Figs. 3(b) and 3(c), we note that the bounds based on mutual information, as in Eq. (2), and information gain are not tight in a quantum scenario, as also anticipated by Eq. (3). For the present protocol, it is possible to show that $\langle I^{(k,l)} \rangle \geq \mathcal{I}_{gain}$ [36]. Despite the 4.5% residual error in the trace distance for the zero mismatch case [Fig. 2(c)], the mutual information (between the system and feedback mechanism) experimentally achieved is very close to its limit, $\langle I^{(k,l)} \rangle = -\sum_l p(l) \ln p(l) = \ln 2$ nats (natural unit of information), as can be observed in Fig. 3(b). As discussed previously the information gain is related to how the system correlates with the memory; hence, it is independent of the control mismatch, which is corroborated by the experimental data in Fig. 3(c).

The kth feedback control operation is designed ideally to map the carbon spin into the equilibrium state $\rho_{\tau_2}^{(eq)}$ of the final Hamiltonian $\mathcal{H}_{\tau_2}^{C}$ (at inverse temperature β) irrespective of the previous nonequilibrium state $ho_{ au_1}$ (produced by the sudden quench). Our aim is to cancel the KL relative entropy, $S_{\text{KL}}(\rho_{\tau_2}^{(k,l)}||\rho_{\tau_2}^{(k,\text{eq})})$, which is successfully achieved for the zero basis mismatch, as can be observed in Fig. 3(d). On other hand, the full dephasing, in the nonunitary feedback, introduces a finite von Neumann entropy variation $\langle \Delta S^{(k,l)} \rangle_{\mathcal{F}}$, see Figs. 3(d) and 3(e). This variation has no energy cost for the demon, since it is a unital process that does not change the working system mean energy. In the framework of the resource theory of quantum thermodynamics, the full dephasing is regarded as a free operation [53,54]. When the control mismatch is increased the final state deviates from the thermal state of $\mathcal{H}_{\tau_2}^{C}$ and consequently the KL relative entropy also increases, as shown in Fig. 3(e).

Discussion.—Employing an information-to-energy trade-off relation, we designed an entropy rectification protocol based on Maxwell's demon. This protocol has been experimentally carried out by a coherent implementation of a measurement-based feedback control on a quantum spin-1/2 system. The demon's memory is a microscopic quantum ancillary system that acquires information through a natural coupling with the working system. Because of the quantum coherence present in our experiment, we have to execute two dephasing operations in order to perform the Maxwell's demon. The first dephasing operation is employed to produce a nonselective measurement, whereas the second is essential to accomplish entropy rectification, canceling the $\langle S_{\text{KL}}(\rho_{\tau_2}^{(k,l)}||\rho_{\tau_2}^{(k,\text{eq})})\rangle$ term in the trade-off relation Eq. (3). The present experiment elucidates the role played by different information quantities in the quantum version of Maxwell's demon. It also provides evidence that the irreversibility on a quantum nonequilibrium dynamics can be mitigated by assessing microscopic information and applying a feed-forward strategy. The approach developed here can be applied to general processes regarding information-to-energy conversion, as for instance, information-based work extraction.

A future experimental challenge would be the investigation of feedback protocols based on generalized quantum measurements and the bounds associated with such a scheme. The analysis and the optimization of the energetic cost for information manipulation by Maxwell's demon, in the quantum scenario, is also an important topic that deserves further attention. From a broad perspective, understanding the trade-off between information and entropy production at the quantum scale might be important to develop applications of quantum technologies with high efficiency.

We thank E. Lutz, M. Ueda, M. Herrera, and I. Henao for valuable discussions. We acknowledge financial support from UFABC, CNPq, CAPES, FAPERJ, and FAPESP. R. M. S. gratefully acknowledges financial support from the Royal Society through the Newton Advanced Fellowship scheme (Grant No. NA140436). This research was performed as part of the Brazilian National Institute of Science and Technology for Quantum Information (INCT-IQ).

P. A. C. and J. P. S. P. contributed equally to this work.

^{*}Corresponding author.

serra@ufabc.edu.br

- R. Kosloff, Quantum thermodynamics, Entropy 15, 2100 (2013).
- [2] J. Goold, M. Huber, A. Riera, L. del Rio, and P. Skrzypczyk, The role of quantum information in thermodynamics—A topical review, J. Phys. A 49, 143001 (2016).
- [3] S. Vinjanampathy and J. Anders, Quantum thermodynamics, Contemp. Phys. 57, 1 (2015).
- [4] J. Millen and A. Xuereb, Perspective on quantum thermodynamics, New J. Phys. 18, 011002 (2016).
- [5] Maxwell's Demon 2: Entropy, Classical and Quantum Information, Computing, edited by H.S. Leff and A.F. Rex (CRC Press, Boca Raton, 2002).
- [6] S. Lloyd, Quantum-mechanical Maxwell's demon, Phys. Rev. A 56, 3374 (1997).
- [7] K. Maruyama, F. Nori, and V. Vedral, Colloquium: The physics of Maxwell's demon and information, Rev. Mod. Phys. 81, 1 (2009).
- [8] M. Plesch, O. Dahlsten, J. Goold, and V. Vedral, Maxwell's Daemon: Information versus particle statistics, Sci. Rep. 4, 6995 (2014).
- [9] E. Lutz and S. Ciliberto, Information: From Maxwell's demon to Landauer's eraser, Phys. Today **68**, 30 (2015).
- [10] C. Bennett, The thermodynamics of computation—a review, Int. J. Theor. Phys. 21, 905 (1982).
- [11] R. Landauer, Irreversibility and heat generation in the computing process, IBM J. Res. Dev. 5, 183 (1961).
- [12] R. Landauer, Dissipation and noise immunity in computation and communication, Nature (London) 335, 779 (1988).
- [13] M. B. Plenio and V. Vitelli, The physics of forgetting: Landauer's erasure principle and information theory, Contemp. Phys. 42, 25 (2001).
- [14] A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, and E. Lutz, Experimental verification of Landauer's principle linking information and thermodynamics, Nature (London) 483, 187 (2012).
- [15] J. M. R. Parrondo, J. M. Horowitz, and T. Sagawa, Thermodynamics of information, Nat. Phys. 11, 131 (2015).

- [16] M. Esposito, U. Harbola, and S. Mukamel, Nonequilibrium fluctuations, fluctuation theorems, and counting statistics in quantum systems, Rev. Mod. Phys. 81, 1665 (2009).
- [17] M. Campisi, P. Hänggi, and P. Talkner, Quantum fluctuation relations: Foundations and applications, Rev. Mod. Phys. 83, 771 (2011).
- [18] C. Jarzynski, Equalities and inequalities: Irreversibility and the second law of thermodynamics at the nanoscale, Annu. Rev. Condens. Matter Phys. 2, 329 (2011).
- [19] P. Hänggi and P. Talkner, The other QFT, Nat. Phys. 11, 108 (2015).
- [20] T. Sagawa, Thermodynamics of Information Processing in Small Systems (Springer Theses, Springer, New York, 2012).
- [21] T. Sagawa and M. Ueda, Information Thermodynamics: Maxwell's Demon in: Nonequilibrium Dynamics (Wiley-VCH, Weinheim, 2013).
- [22] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki, and M. Sano, Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality, Nat. Phys. 6, 988 (2010).
- [23] E. Roldán, I. A. Martínez, J. M. R. Parrondo, and D. Petrov, Universal features in the energetics of symmetry breaking, Nat. Phys. 10, 457 (2014).
- [24] J. V. Koski, V. Maisi, T. Sagawa, and J. P. Pekola, Experimental observation of the role of mutual information in the nonequilibrium dynamics of a Maxwell demon, Phys. Rev. Lett. **113**, 030601 (2014).
- [25] J. V. Koski, V. F. Maisi, J. P. Pekola, and D. V. Averin, Experimental realization of a Szilard engine with a single electron, Proc. Natl. Acad. Sci. U.S.A. 111, 13786 (2014).
- [26] J. V. Koski, A. Kutvonen, I. M. Khaymovich, T. Ala-Nissila, and J. P. Pekola, On-Chip Maxwell's Demon as an Information-Powered Refrigerator, Phys. Rev. Lett. 115, 260602 (2015).
- [27] M. D. Vidrighin, O. Dahlsten, M. Barbieri, M. S. Kim, V. Vedral, and I. A. Walmsley, Photonic Maxwell's Demon, Phys. Rev. Lett. **116**, 050401 (2016).
- [28] J. P. S. Peterson, R. S. Sarthour, A. M. Souza, I. S. Oliveira, J. Goold, K. Modi, D. O. Soares-Pinto, and L. C. Céleri, Experimental demonstration of information to energy conversion in a quantum system at the Landauer limit, Proc. R. Soc. A 472, 20150813 (2016).
- [29] M. A. Ciampini, L. Mancino, A. Orieux, C. Vigliar, P. Mataloni, M. Paternostro, and M. Barbieri, Experimental entanglement-enhanced work extraction based on a Maxwell's demon, arXiv:1601.06796.
- [30] M. H. Levitt, Spin Dynamics: Basics of Nuclear Magnetic Resonance, 2nd ed. (Wiley, New York, 2008).
- [31] S. Oliveira, T. J. Bonagamba, R. S. Sarthour, J. C. C. Freitas, and R. R. deAzevedo, *NMR Quantum Information Processing* (Elsevier, Amsterdam, 2007).
- [32] J. A. Jones, Quantum computing with NMR, Prog. Nucl. Magn. Reson. Spectrosc. 59, 91 (2011).
- [33] T. B. Batalhão, A. M. Souza, L. Mazzola, R. Auccaise, R. S. Sarthour, I. S. Oliveira, J. Goold, G. De Chiara, M. Paternostro, and R. M. Serra, Experimental Reconstruction of Work Distribution and Study of Fluctuation Relations in a Closed Quantum System, Phys. Rev. Lett. 113, 140601 (2014).
- [34] T. B. Batalhao, A. M. Souza, R. S. Sarthour, I. S. Oliveira, M. Paternostro, E. Lutz, and R. M. Serra, Irreversibility

and the Arrow of Time in a Quenched Quantum System, Phys. Rev. Lett. **115**, 190601 (2015).

- [35] We have considered unital processes in order to obtain the fluctuation relation in Eq. (1). It is worth mentioning that this consideration is not necessary for obtaining the trade-off relation introduced in Eq. (3).
- [36] See Supplemental Material at http://link.aps.org/ supplemental/10.1103/PhysRevLett.117.240502, which includes Ref. [37].
- [37] P. Talkner, E. Lutz, and P. Hänggi, Fluctuation theorems: Work is not an observable, Phys. Rev. E 75, 050102(R) (2007).
- [38] A. E. Rastegin, Non-equilibrium equalities with unital quantum channels, J. Stat. Mech. (2013) P06016.
- [39] T. Albash, D. A. Lidar, M. Marvian, and P. Zanardi, Fluctuation theorems for quantum processes, Phys. Rev. E 88, 032146 (2013).
- [40] T. Sagawa and M. Ueda, Generalized Jarzynski Equality under Nonequilibrium Feedback Control, Phys. Rev. Lett. 104, 090602 (2010).
- [41] T. Sagawa and M. Ueda, Fluctuation Theorem with Information Exchange: Role of Correlations in Stochastic Thermodynamics, Phys. Rev. Lett. 109, 180602 (2012).
- [42] Y. Morikuni and H. Tasaki, Quantum Jarzynski-Sagawa-Ueda Relations, J. Stat. Phys. 143, 1 (2011).
- [43] H. J. Groenewold, A problem of information gain by quantal measurements, Int. J. Theor. Phys. 4, 327 (1971).
- [44] G. Lindblad, An entropy inequality for quantum measurements, Commun. Math. Phys. 28, 245 (1972).
- [45] M. Ozawa, On information gain by quantum measurements of continuous observables, J. Math. Phys. 27, 759 (1986).
- [46] C. A. Fuchs and K. Jacobs, Information-tradeoff relations for finite-strength quantum measurements, Phys. Rev. A 63, 062305 (2001).
- [47] F. Buscemi, M. Hayashi, and M. Horodecki, Global Information Balance in Quantum Measurements, Phys. Rev. Lett. 100, 210504 (2008).
- [48] M. Berta, J. M. Renes, and M. M. Wilde, Identifying the information gain of a quantum measurement, IEEE Trans. Inf. Theory 60, 7987 (2014).
- [49] K. Jacobs, *Quantum Measurement Theory and its Application* (Cambridge University Press, Cambridge, England, 2014).
- [50] K. Funo, Y. Watanabe, and M. Ueda, Integral quantum fluctuation theorems under measurement and feedback control, Phys. Rev. E 88, 052121 (2013).
- [51] T. Sagawa, in Second Law-Like Inequalities with Quantum Relative Entropy: An Introduction in:, edited by M. Nakahara and S. Tanaka, Lectures on Quantum Computing, Thermodynamics and Statistical Physics (World Scientific, Singapore, 2012).
- [52] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, 10th anniversary edition. (Cambridge University Press, Cambridge, England, 2000).
- [53] M. Lostaglio, D. Jennings, and T. Rudolph, Description of quantum coherence in thermodynamic processes requires constraints beyond free energy, Nat. Commun. 6, 6383 (2015).
- [54] K. Korzekwa, M. Lostaglio, J. Oppenheim, and D. Jennings, The extraction of work from quantum coherence, New J. Phys. 18, 023045 (2016).