Dark Energy after GW170817 and GRB170817A

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The observation of GW170817 and its electromagnetic counterpart implies that gravitational waves travel at the speed of light, with deviations smaller than a few $10^{-15}$. We discuss the consequences of this experimental result for models of dark energy and modified gravity characterized by a single scalar degree of freedom. To avoid tuning, the speed of gravitational waves must be unaffected not only for our particular cosmological solution but also for nearby solutions obtained by slightly changing the matter abundance. For this to happen, the coefficients of various operators must satisfy precise relations that we discuss both in the language of the effective field theory of dark energy and in the covariant one, for Horndeski, beyond Horndeski, and degenerate higher-order theories. The simplification is dramatic: of the three functions describing quartic and quintic beyond Horndeski theories, only one remains and reduces to a standard conformal coupling to the Ricci scalar for Horndeski theories. We show that the deduced relations among operators do not introduce further tuning of the models, since they are stable under quantum corrections.

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Introduction.—The association of the GW170817 [1] and GRB170817A [2] events allowed one to make an extraordinarily precise measurement of the speed of gravitational waves (GWs): it is compatible with the speed of light with deviations smaller than a few $10^{-15}$ [3]. This measurement dramatically improves our understanding of dark energy and modified gravity. These scenarios are characterized by a cosmological “medium” which interacts gravitationally with the rest of matter. This medium, at variance with a simple cosmological constant, spontaneously breaks Lorentz invariance so that there is no a priori reason to expect that gravitational waves, which are an excitation of this medium, travel at the same speed as photons [4,5].

The measurement is of particular relevance since it probes the speed of GWs over cosmological distances. The change of speed might be locally reduced in high density environments, but it is difficult to believe that this screening effect can persist over distances of order 40 Mpc. Moreover, one has to stress that this is a low-energy measurement, at a scale as low as 10 000 km. For such a low energy, one should be allowed to use the effective field theory (EFT) of dark energy or modified gravity which applies to cosmological scales. Actually, in the theories we are going to study, the cutoff may be of the same order as the measured GW momentum and high-dimension operators may play some role; however, one does not expect that high-energy corrections conspire to completely cancel the modification of the GW speed. On the other hand, previous stringent limits from gravitational Cherenkov radiation of cosmic rays [6] are only applicable to high-energy GWs, well outside the regime of validity of the EFTs describing dark energy and modified gravity. Moreover, these bounds only apply to GWs traveling faster, and not slower, than light. For other limits, see Refs. [7–10].

With these caveats in mind, in this Letter we want to explore what the consequences are of this measurement in the context of the EFT of dark energy [11–13] and in its covariant counterpart, the Horndeski [14,15] and the beyond Horndeski theories [16] (see, also, Ref. [17]). If we impose that the absence of an effect is robust under tiny variations of the cosmological history—say, a small variation of the dark matter abundance—we find that one needs precise relations among the various coefficients of the operators. This allows us to derive the most general scalar-tensor theory compatible with GWs traveling at the speed of light. Since the required relations must be satisfied with great accuracy, given the experimental precision, one needs to understand whether they are radiatively stable. We will see that they are stable under quantum corrections due to the nonrenormalization properties of these theories.

Consequences for the EFT of dark energy.—The EFT of dark energy is a convenient way to parametrize cosmological perturbations around a Friedmann-Robertson-Walker (FRW) solution with a preferred slicing induced by a time-dependent background scalar field. For the time being, we assume that matter is minimally coupled to the gravitational metric; we will come back to this point later on.

Expanded around a FRW background $ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$ and written in a gauge where the time coincides with uniform field hypersurfaces, the EFT action reads

$$S = \int d^4 x \sqrt{-g} \left[ \frac{M_4^2}{2} f^{(4)} R - \Lambda - cg^{00} + \frac{m_s^4}{2} (\delta g^{00})^2 ight. $$

$$\left. - \frac{m_s^2}{2} \delta K \delta g^{00} - m_s^2 \delta \mathcal{K}_2 + \frac{m_s^4}{2} \delta g^{00} R - \frac{m_s^2}{2} \delta g^{00} \delta \mathcal{K}_2 - \frac{m_s^6}{3} \delta \mathcal{K}_3 - \frac{m_s^2}{3} \delta g^{00} \delta \mathcal{K}_2 - \frac{m_s^4}{3} \delta g^{00} \delta \mathcal{K}_3 \right].$$

(1)
Here, \((4)R\) is the 4D Ricci scalar, \(\delta g^{00} = 1 + g^{00}\), \(\delta K^a_\mu \equiv K^a_\mu - H^a_\mu\), is the perturbation of the extrinsic curvature of the time hypersurfaces \((H \equiv \dot{a}/a)\), \(R^a_\mu\) is the 3D Ricci tensor of these hypersurfaces, and \(\delta K\) and \(R\) are, respectively, their trace. For convenience, we have also defined

\[
\delta K_2 \equiv \delta K^2 - \delta K^a_\mu \delta K^a_\mu, \quad \delta G_2 \equiv \delta K^a_\mu R^a_\mu - \delta KR/2, \\
\delta K_3 \equiv \delta K^3 - 3\delta K^a_\mu \delta K^a_\mu + 2\delta K^a_\mu \delta K^a_\mu.
\]

(2)

While \(M_4^2\) is constant, the other parameters are time-dependent functions. As we will discuss in the following section, this action describes the cosmological perturbations in Horndeski (for \(\tilde{m}_4^2 = m_4^2\) and \(\tilde{m}_6 = m_6\)) and beyond Horndeski theories. At quadratic order, it has been introduced in Ref. [18]. At higher order, we have written only the operators that contribute to the leading number of spatial derivatives. These dominate the nonlinear regime of structure formation and the Vainshtein regime (see, e.g., Refs. [19–21] for details). At quintic or higher order, there are no such operators. The other operators present in Horndeski and beyond Horndeski theories are not explicitly written but will be discussed below. More general higher-order operators will be considered below.

In Eq. (1), GWs only enter in the 4D and 3D Ricci tensor and in the trace-free part of \(K^a_\mu\). At quadratic order, the operator \(m_6^2 \delta K_2\) contributes to the graviton kinetic energy, changing the normalization of the effective Planck mass—which becomes \(M^2 \equiv M^2 + 2m_4^2\)—modifying the propagation speed of gravitational waves \([18,22]\),

\[
c_7^2 - 1 = -2m_4^2/M^2.
\]

(3)

(Notice that \(m_4^2\) can have either sign; it is written as a square just to keep track of dimensions.) Thus, the constraint of GW170817 implies that the coefficient of the operator \(m_4^2 \delta K_2\) must be extremely small,

\[
m_4^2 = 0.
\]

(4)

However, the value of this parameter depends on the particular background the EFT is expanded around. In particular, by changing by a tiny amount the Hubble expansion or the background energy density of the scalar (or, correspondingly, the dark matter abundance), the coefficients of the EFT action get reshuffled. A change in the background appears in the EFT action as a background value for \(\delta g^{00}\) and \(\delta K\). To robustly set to zero \(m_4^2\), we should set to zero also all those operators that can generate it by a small change of the background solution. As an example, consider \(m_7^2 \delta g^{00} \delta K_2\). When \(\delta g^{00}\) is evaluated on the background, this operator becomes quadratic and shifts the parameter \(m_4^2\); i.e., \(\delta m_4^2 = m_7^2 \delta g^{00}\) \(\delta K_2\). However, the change in \(c_7^2\) can be compensated by the operator \(m_7^2 \delta g^{00} R\) if \(m_4^2\) is chosen appropriately. By choosing

\[
\tilde{m}_4^2 = m_4^2 \quad (= 0 \text{ in Horndeski}),
\]

these two operators combine to change the overall normalization of the graviton action, keeping the graviton on the light cone. (In Horndeski, \(m_4^2 = \tilde{m}_4^2 = 0\).) The same tuning must hold for operators with more powers of \(\delta g^{00}\) that have not been explicitly included in the action, such as \((\delta g^{00})^2 R\), \((\delta g^{00})^2 \delta K_2\), etc.

Let us consider the remaining operators, starting with \(m_6 \delta K_3\). When one of the \(\delta K^a_\mu\) or \(\delta K\) in the cubic expression for \(\delta K_3\) is evaluated on the background, this operator becomes quadratic and contributes to \(m_4^2\). Using \(\delta K^a_\mu\) \(\text{bkgd} = \delta H_{\text{bkgd}} \delta g_{\mu0}\), one finds \(\delta m_4^2 = \delta H_{\text{bkgd}} m_6\). Notice that the dependence on the background is through \(\delta H_{\text{bkgd}}\) and not through \(\delta g_{00}\) \(\text{bkgd}\), so that its contribution cannot be compensated by either \(\tilde{m}_4^2\) or \(m_4^2\). It is easy to get convinced that the same happens for \(m_6\) and \(m_7\). When \(\delta g^{00}\) \(\text{bkgd}\) is evaluated on the background, upon use of Eq. (8) of Ref. [18], one finds that the operator \(m_6\) shifts \(m_4^2\) by \(\delta m_4^2 = -\frac{1}{2}(\tilde{m}_6 \delta g^{00})\bkgd\). Finally, the operator \(m_7\) induces \(\delta m_4^2 = m_7 \delta g_{00} \bkgd \delta K_{bkgd}\). Since the background enters differently in all these operators, they must be precisely set to zero,

\[
m_6 = \tilde{m}_6 = m_7 = 0.
\]

(6)

As we will discuss below, the relations we found are stable under radiative corrections.

**Covariant action.**—Let us see how the constraints of GW170817 on the EFT of dark energy translate for covariant theories. In particular, we consider the action

\[
S = \int d^4x \sqrt{-g} \sum L_i,
\]

(7)

where we have defined the Lagrangians

\[
L_2 \equiv G_2(\phi, X), \quad L_3 \equiv G_3(\phi, X) \square \phi, \\
L_4 \equiv G_4(\phi, X) (4)R - 2G_{4X}(\phi, X)(\Box \phi^2 - \phi_{\mu\nu} \phi_{\mu\nu}) + F_4(\phi, X) e^{\mu\nu\rho\sigma} e^{\rho\sigma\ell\eta} \phi_{\mu\nu} \phi_{\ell\eta} \phi_{\rho\sigma}, \\
L_5 \equiv G_5(\phi, X) (4)G_{\mu\nu} \phi_{\mu\nu} + \frac{1}{3} G_{5X}(\phi, X)(\Box \phi^2 - 3 \Box \phi_{\mu\nu} \phi_{\mu\nu} + 2 \phi_{\mu\nu} \phi_{\mu\nu} \phi_{\rho\sigma}), \\
F_5(\phi, X) e^{\mu\nu\rho\sigma\ell\eta} e^{\rho\sigma\ell\eta\mu\nu} \phi_{\mu\nu} \phi_{\ell\eta} \phi_{\rho\sigma} \phi_{\ell\eta}
\]

(8)

depend that on a scalar field \(\phi, X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi\) and second derivatives of the field. For convenience, we denote the scalar field derivatives by \(\phi_{\mu\nu} \equiv \nabla_\mu \phi, \phi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \phi, \) and \(\Box \phi \equiv \phi_{\mu\nu}\). The symbol \(e_{\mu\nu\rho\sigma}\) is the totally antisymmetric Levi-Civita tensor, and a comma denotes a partial derivative with respect to the argument. Horndeski theories are recovered by the conditions \(F_4(\phi, X) = 0\) and \(F_5(\phi, X) = 0\), which guarantee that the equations of motion are purely second order. If \(L_5 = 0\) and \(G_4 - 2XG_{4X} \neq 0\) \((L_4 = 0 \) and
which can be obtained by imposing that both Lagrangians are generated by the same disformal transformation \[25\]. In summary, the quartic and quintic Lagrangians of beyond Horndeski theories are described in terms of three independent functions of \(\phi\) and \(X\).

To compare with the EFT approach, let us write the relevant parameters in Eq. (1) in terms of the covariant functions \(G_4\), \(G_5\), \(F_4\), and \(F_5\) above (of course, \(L_2\) and \(L_3\) do not affect GWs)

\[
M^2 = 2G_4 - 4XG_{4,X} - X(G_{5,\phi} + 2H\phi G_{5,X})
+ 2X^2 F_4 - 6H\phi X^2 F_5,
\]

\[
m_4^2 = \tilde{m}_4^2 + X^2 F_4 - 3H\phi X^2 F_5,
\]

\[
\tilde{m}_4^2 = -[2XG_{4,X} + XG_{5,\phi} + (H\phi - \phi)XG_{5,X}],
\]

\[
m_5^2 = X[2G_{4,X} + 4XG_{4,X} + H\phi(3G_{5,X} + 2XG_{5,X})
+ G_{5,\phi} + XG_{5,\phi} - 4XF_4 - 2X^2 F_4 X
+ H\phi X(15F_5 + 6XF_{5,X})],
\]

\[
m_6 = \tilde{m}_6 - 3\phi X^2 F_5, \quad \tilde{m}_6 = -\phi XG_{5,X},
\]

\[
m_7 = \frac{1}{2} \phi X(3G_{5,X} + 2XG_{5,X} + 15XF_5 + 6X^2 F_5,X).
\]

Setting the speed of GWs to 1, i.e., Eq. (4), implies that the particular combination appearing in the expression of \(m_7^2\) above vanishes. This must be true on any background and, thus, must hold for any value of \(\phi\), \(H\), and \(\phi\) (or \(X\)). This implies, respectively,

\[
G_{5,X} = 0, \quad F_5 = 0, \quad 2G_{4,X} - XF_4 + G_{5,\phi} = 0.
\]

for any \(X\) and \(\phi\). Thus, \(G_5\) can be at most a function of \(\phi\), the beyond Horndeski term \(F_5\) must be absent, and there is a relation between \(G_{4,X}\) and \(F_4\) and their derivatives. The first two conditions automatically imply Eq. (6). It is also straightforward to verify that Eq. (5) is a consequence of Eq. (11). Finally, using Eq. (11) in \(L_4\) and \(L_5\) of the Lagrangians (8), after some manipulations and integrations by parts, we remain with

\[
L_{c_1 = 1} = B_2(\phi, X) + B_3(\phi, X)\Box \phi + B_4(\phi, X)^{(4)R}
- \frac{4}{X} B_{4,X}(\phi, X)(\phi^\mu \phi^\mu \Box \phi - \phi^\mu \phi^\nu \phi^\rho \phi^\rho)\).
\]
Let us assume the functions $G_4$ and $G_5$ do not depend on $\phi$ and are of the form

$$G_4(X) = \frac{\Lambda_8}{\Lambda_5^2} \tilde{G}_4 \left( \frac{X}{\Lambda_5^2} \right), \quad G_5(X) = \frac{\Lambda_8}{\Lambda_3^2} \tilde{G}_5 \left( \frac{X}{\Lambda_3^2} \right).$$

To have sizeable dark energy effects, one takes $\Lambda_2 \sim (M_{\text{Pl}}H_0)^{1/2}$ and $\Lambda_3 \sim (M_{\text{Pl}}H_0^2)^{1/3}$, where $M_{\text{Pl}}$ is the Planck mass. We take the dimensionless functions $\tilde{G}$ to be polynomials in their variable with order-one coefficients $c_n$. The result of Ref. [26] is that all these coefficients are corrected by a relative amount of order $\delta c_n \sim (\Lambda_3/\Lambda_2)^4 \sim 10^{-40}$. This is much smaller than the $10^{-15}$ cancellation implied by the measurement of the speed of GWs: it is completely negligible unless one goes to extraordinary large $n$. The same conclusions can be obtained in a beyond Horndeski theory [28]. In conclusion, the relations one has to invoke to be compatible with GW170817 are technically natural in the sense that once imposed at tree level, they are stable under quantum corrections.

Higher-order operators and conformal transformations.—It was recently pointed out that there are more general theories than those in Eq. (8) that do not propagate additional degrees of freedom [24]. In the EFT language, they give rise to particular combinations of the quadratic operators [29]

$$\begin{align*}
\delta \mathcal{L} = \int d^4x \sqrt{-g} \frac{M^2}{2} \left( -\frac{2}{3} \alpha g_1 \delta K^2 + 4 \beta_1 \delta KV + \beta_2 V^2 + \beta_3 a_i a^i \right),
\end{align*}$$

where $V \equiv -\frac{1}{2} (\dot{\phi}^2 - N^2 \partial \phi^0 \partial \phi^0)/g^{00}$ and $a_i = -\frac{1}{2} \partial^i g^{00}/g^{00}$. It is straightforward to see that these operators do not affect the speed of GWs. This is true around the given background but also if one considers different backgrounds: since these operators have two derivatives, only $\delta \phi^{00}$ can be turned on, but it is easy to see that even around the new background, GWs are unaffected.

In the covariant language, these theories can be obtained starting from beyond Horndeski and performing a conformal transformation that depends on $X$. Since this does not change the light cone, if one starts from the action (12) also the resulting degenerate higher-order theories will not affect GWs’ speed of propagation. Under a general conformal transformation $g_{\mu\nu} \rightarrow C(\phi, X) g_{\mu\nu}$ [30,31], we find (we assume $C$ is not linear in $X$)

$$\begin{align*}
L_{c_T=1} &= \tilde{B}_2 + \tilde{B}_3 \Box \phi + CB_4^{(4)} R - \frac{4CB_4^{(4)}X}{X} \phi^\rho \phi^\nu \phi_{\mu\nu} \Box \phi \\
&\quad + \left( \frac{4CB_4^{(4)}X}{X} + 6B_2 C X^2 + 8CB_4^{(4)}X \right) \phi^\rho \phi^\nu \phi_{\mu\nu} \phi_{\rho\nu} \\
&\quad - \frac{8C X B_4^{(4)}X}{X} (\phi_{\mu\nu} \phi_{\rho\nu})^2.
\end{align*}$$

(We do not explicitly show the expression of $\tilde{B}_2$ and $\tilde{B}_3$, since they are anyway free functions unrelated to the other terms.)

This is the most general degenerate theory which can be obtained from Horndeski by a metric redefinition compatible with $c_T^2 = 1$. In the classification of Ref. [24], it belongs to type Ia degenerate higher-order scalar-tensor theories.

There are theories in which spatial (but not time) higher derivatives are present and, therefore, do not propagate extra degrees of freedom. In the case of the ghost condensate [32], the modification of the GW speed goes as $c_T^2 - 1 \sim \frac{M_{GC}^2}{M_{Pl}^2}$, where $M_{GC}$ is the typical scale of the model. Since experimental bounds on the modification of the Newton law give $M_{GC} \lesssim 10$ MeV, one does not expect any significant effect on the speed of GWs. On the other hand, in the case of Einstein-aether [33] and Hořava gravity [34], $c_T$ is expected to deviate from unity, and the bound of GW170817 represents a severe constraint on these models.

Disformal transformations.—So far, we have assumed that matter is minimally coupled to the metric. There is no lack of generality in this, provided there is a universal coupling for all matter species, since one can always go to this frame with a suitable conformal and disformal transformation. In this frame, the results of GW170817 imply that GWs must travel on the light cone of the metric. If one chooses to go to a different disformal frame, both matter and GWs will acquire a common disformal coupling: since they both travel at the same speed, this is obviously still compatible with what LIGO and Virgo observed. In the new frame, the gravitational action will not be of the form (12) or (16). For example, one can decide to disform the beyond Horndeski theories (12) to become a Horndeski theory, but now both GWs and light will not move on the geodesics of the metric.

Conclusion.—We have obtained the most general scalar-tensor theories propagating a single scalar degree of freedom compatible with the observation of GW170817. In the Jordan frame, the parameters of the EFT of dark energy of these theories must satisfy Eqs. (4)–(6). Analogous relations must be imposed on the operators containing higher-order terms in $\delta \phi^{00}$. The most general covariant theory is given by Eq. (16).

After GW170817, quartic and quintic Horndeski theories are excluded, unless they reduce to a standard conformal coupling to $(\phi)^4 R$. Consequently, the cubic and quartic operators of Eq. (1) must be absent, which implies that the Vainshtein mechanism allowed by them [19] cannot take place (screening must rely only on the cubic theories) and that no signatures of these nonlinear operators should be found in the large-scale structures (see, e.g., Ref. [35]). For beyond Horndeski theories, the Vainshtein mechanism is broken inside compact bodies [20]. We leave for the future to study what consequence this has on the theories (16).

The relations that need to be satisfied are technically natural, but it would be nice to investigate whether they can be derived from some underlying symmetry. On the experimental side, further observations over a larger distance and at lower frequencies will make the limits even more robust to Vainshtein screening and higher derivative corrections.
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Note added in the proof.—Other articles [36–38] whose content overlaps with ours appeared recently.