Single-Atom Demonstration of the Quantum Landauer Principle

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(Received 9 February 2018; revised manuscript received 10 March 2018; published 21 May 2018; corrected 16 July 2019)

One of the outstanding challenges to information processing is the eloquent suppression of energy consumption in the execution of logic operations. The Landauer principle sets an energy constraint in deletion of a classical bit of information. Although some attempts have been made to experimentally approach the fundamental limit restricted by this principle, exploring the Landauer principle in a purely quantum mechanical fashion is still an open question. Employing a trapped ultracold ion, we experimentally demonstrate a quantum version of the Landauer principle, i.e., an equality associated with the energy cost of information erasure in conjuction with the entropy change of the associated quantized environment. Our experimental investigation substantiates an intimate link between information thermodynamics and quantum candidate systems for information processing.

DOI: 10.1103/PhysRevLett.120.210601

It was Landauer who expostulated, for the first time, a minimum amount of energy required to be consumed in deletion of a classical bit of information, known as the Landauer principle (LP) [1], implying an irreversibility of logical operations [2–4]. In terms of the LP, the erasure of information is fundamentally a dissipative process, which dissipates at least $k_B T \ln 2$ amount of heat, called the Landauer bound, from the system into the attached reservoir, where $k_B$ and $T$ represent the Boltzmann constant and reservoir temperature, respectively. On the other hand, if we try to understand violation of the second law of thermodynamics by Maxwell’s demon (an intellectual creature envisioned by Maxwell), who converts thermal energy of the reservoir into useful work [5–7], the entropy cost should be considered regarding the demon’s memory, which is also subject to the LP. So far, much of the discussion has been devoted to the validity and usefulness of the LP, including some skepticism and misunderstanding [8–12]. Since suppression of energy dissipation to a possible minimum is indispensable towards the continued development of digital computers [13,14], even for quantum information processing [15], further investigation of the LP is certainly of particular importance.

Some experimental attempts, subject to the conventional inequality of the LP, have been carried out to approach the Landauer bound using single-bit operations [16–20]. The latest experiment [20], for example, was implemented on a nanoscale digital magnetic memory bit, by which the intrinsic energy dissipation per single-bit operation was detected. In contrast, a very relevant investigation for extracting $k_B T \ln 2$ of heat in the creation of one bit of information [21], which is the complementary study of the LP in the context of a Szilárd engine [22]—a quantitative Maxwell demon—was accomplished recently on a single-electron transistor. However, all these aforementioned considerations surmise that the heat related to the removal or generation of a bit of information is regarding an open environment, which is a reservoir with much bigger size than the system. If we extend the treatments to a quantum domain, e.g., the information to be erased being encoded in a qubit and the environment being quantized, the model needs to be substantially reconsidered. The entropy of a quantum system is associated with the characteristic of the state, rather than the thermodynamic arrow of time. Additionally, in contrast to a large reservoir considered in the original version of the LP, the quantized reservoir is of a finite size and hence vacillating in interaction with the qubit system under quantum operations for information erasure. So, different from the previous assumption of final product states [23–25], quantum correlation appears between the system and the quantized reservoir during a quantum erasure process, which requires evaluation via mutual information and relative entropy. However, even with
elaborately designed systems under the consideration of a quantized reservoir, experimentally measuring mutual information and relative entropy is not easy to accomplish [26,27], which hampers efforts to exactly approach the Landauer bound.

Here we report an experimental investigation of a quantum mechanical LP by an experimental evaluation of system-reservoir correlation and entropy change during the erasure process. Our operations are based on a single ultracold $^{40}\text{Ca}^+$ ion confined in a linear Paul trap. Trapped ultracold ions, with the possibility of precise manipulation, have been considered as an ideal platform to explore the thermodynamics in a quantum domain with ultimate accuracy [28–31]. For our purpose, we consider the two internal levels of the ion as the qubit system and the vibrational degree of freedom of the ion as a finite-temperature reservoir. Removing information encoded initially in the qubit, we perceive the quantized LP by observing the phonon number variation in the quantized internal levels of the ion as the qubit system and the reservoir states turn to a mixed state resulting from an erasure process. Our operations are based on a single quantum mechanical LP by an experimental evaluation of the states $|\downarrow\rangle$ and $|\uparrow\rangle$, indicating that the initial entropy of the system is superlative, but in contrast, our available information about the system is minimum; that is, we have no explicit information about the existing state of the ion. For a general case, we consider the reservoir to be initially in a thermal state with an associated energy defined as $E_0 = \text{Tr}[H_R\rho_R]$ for a given state $\rho_R$ [see Fig. 1(b)]. The erasure is performed by employing a red-sideband transition, governed by $H_R = \eta \hbar \Omega (\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi})/2$, where $\Omega$ and $\phi$ represent the Rabi frequency and laser phase, respectively, $\sigma_+$ ($\sigma_-$) indicates the raising (lowering) operator of the spin, and $\sigma$ ($\sigma'$) denotes the annihilation (creation) operator of the quantized vibration of the ion in the $z$ direction. The erasure process produced by $U_R = \exp(-iH_R t)$, with $t$ as the pulse length, causes the population transfer from $|\uparrow\rangle$ to $|\downarrow\rangle$, corresponding to an entropy decrease of the system, which is accompanied by the reservoir energy increase up to $E_{\text{res}}$, as depicted in Fig. 1(b2). The erasure process ends with the system in the polarized state $|\downarrow\rangle$ and the reservoir changes to the final state $\rho_R$, the energy of which is given as $E_j = \text{Tr}[H_R\rho'_R]$; see Fig. 1(b3). So we finally have a complete enlightenment about the state of the system accomplished by virtue of the heat increment $\Delta Q = E_j - E_0$. In this way, we may verify Eq. (1) by observing the variation of the population and phonon number in the qubit and the reservoir, respectively, where the entropy decrease of a system is $\Delta S = \ln 2$ in an ideal erasure for the system attached to a zero-temperature reservoir. The corresponding experimental procedure is drawn in Fig. 1(c).

In our actual implementation, superposition of the qubit states was achieved by a carrier-transition operator $U_C(\theta_0, 0) = \cos(\theta_0/2)I - i \sin(\theta_0/2)\sigma_z$, where $\sigma_z$ is the Pauli operator. Since the dephasing rate ($\sim 0.5$ kHz) of the
In Figs. 2(a) and 2(b) we show, respectively, the absolute and relative entropy of the system. Therefore, we estimated the mutual information by direct measurement of the mutual information is complicated due to its relevance to the system-reservoir entanglement. Therefore, we estimated the mutual information by an alternative way, as explained in Ref. [34], instead of direct experimental measurements.

With the data measured above, the Landauer bound and the improved LP are witnessed under the different reservoir temperatures in Fig. 2(c). Our observations clearly exhibit that the equality in the original form of the LP cannot be accessed quantum mechanically at the finite temperature. In contrary to the classical version of the LP in which the huge reservoir changes very little during the erasure process such that the mutual information and the relative entropy of the

The phonon number due to heat variation, as plotted in Fig. 2(c), could be calculated by Eq. (S14) in Ref. [34], and meanwhile the relative entropy of the reservoir is acquired. However, the populations in $|\downarrow\rangle$ and $|\uparrow\rangle$ at the end of the erasure are monitored from the 397-nm spontaneous-emission fluorescence spectrum [see Fig. 2(d)], which yields a variation of the system entropy. Moreover, the mutual information term is smaller with respect to other terms in Eq. (1), and a direct measurement of the mutual information is complicated due to its relevance to the system-reservoir entanglement. Therefore, we estimated the mutual information by an alternative way, as explained in Ref. [34], instead of direct experimental measurements.
reservoir can be ignored, both states of the system and reservoir in the quantum regime vary during the erasure process, which yields a quantum correlation between the system and the reservoir. Intrinsically, we can observe a large disparity between the heat consumption of the reservoir (black curves) and the entropy decrease of the system (blue curves), due to the difference from quantum correlation and the relative entropy of the reservoir. This difference turns out to be larger at lower temperature because in this case the relative entropy becomes more dominant in Eq. (1). In particular, in the limit of zero temperature, one may find the system’s entropy decrease $\Delta S = \ln 2$, the mutual information $I(S':R') = 0$, and the relative entropy $D(\rho_R' || \rho_R) \to \infty$, which agrees with the result of $1/T \to \infty$ in the left-hand side of Eq. (1). In this scenario, the nearly perfect overlap between the red dashed and black solid curves implies that Eq. (1) provides a better way to understand the LP and the associated Landauer bound in the quantum regime.

It is interesting to note that the quantum LP is very sensitive to the initial condition of the model. Figure 3 presents the variation of the system entropy from negative to positive and then to negative again, arguing that entropy decrease, i.e., removal of information, occurs in the system for $\theta_c \in [0.54(3), 2.80(2)]$, and otherwise, the system entropy increases during the erasure, indicating creation of information. This indicates that our implementation could demonstrate both the generation and deletion of the system’s information by simply tuning $\theta_c$ (i.e., varying the initial state of the system). Particularly, the generation of information in the region of $\theta_c < 0.54(3)$ corresponds to a fully quantized single-qubit Szilárd engine, the opposite process of the quantum LP. However, the observation in the region of $\theta_c > 2.80(2)$ is counterintuitive, where the generation of information is accompanied by the energy increase of the reservoir. This phenomenon is completely different from the aforementioned quantum LP and Szilárd engine, but reflects the significant role of the relative
entropy of the reservoir played in the fully quantized system-reservoir model [37].

In summary, we have demonstrated, in a more pertinent way, the first experimental investigation of the quantum mechanical LP using a single ultracold trapped ion. This is an imperative step towards better understanding of the fundamental physical limitations of irreversible logic operations at the quantum level. Our observation confirms that the LP still holds even at the quantum level after some modification by introducing quantum information quantities. Our experimental evidence might be helpful for efficient initialization of a future quantum computer involving an artificial quantum reservoir for fast elimination of the encoded information from large numbers of qubits, in which the information erasure yields more heat consumption than the classical counterpart due to system-reservoir correlations. For the same reason, quantum error correction in such a quantum computer would also cost higher heat (or energy). We believe that the experiment reported here will open an avenue towards further exploration of the Landauer bound in the quantum regime as well as possible applications.

This work was supported by National Key Research & Development Program of China under Grant No. 2017YFA0304503, by National Natural Science Foundation of China under Grants No. 11734018, No. 11674360, No. 11404377, No. 91421111, No. 11571313, and No. 11574353, and by the Strategic Priority Research Program of the Chinese Academy of Sciences under Grant No. XDB21010100, and by the Youth Innovation Promotion Association of the Chinese Academy of Sciences under Grant No. 2016299. T. P. X. was partially supported by Guangxi Natural Science Foundation of China (2015GXNSFAA139307), Guangxi Key Laboratory of Trusted Software (kk201505), Program for Innovative Research Team of Guilin University of Electronic Technology. K. R. thankfully acknowledges support from CAS-TWAS president’s fellowship.

L. L. Y., T. P. X., and K. R. contributed equally to this work.

Note added.—Recently we became aware of a previous work [38] which presented an equality to be equivalent to Eq. (1) above, although no connection to Landauer principle was made explicitly there.

Correction: The omission of a support statement in the Acknowledgment section has been fixed.