Editors' Suggestion Featured in Physics

Theory of Disorder-Induced Half-Integer Thermal Hall Conductance

David F. Mross, Yuval Oreg, Ady Stern, Gilad Margalit, and Moty Heiblum
Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

(Received 22 January 2018; revised manuscript received 24 April 2018; published 9 July 2018)

The thermal Hall conductance in the half-filled first Landau level was recently measured to take the quantized noninteger value $\kappa_{xy} = 5/2$ (in units of temperature times $\pi^2 k_B^2/3h$), which indicates a non-Abelian phase of matter. Such exotic states have long been predicted to arise at this filling factor, but the measured value disagrees with numerical studies, which predict $\kappa_{xy} = 3/2$ or $7/2$. We resolve this contradiction by invoking the disorder-induced formation of mesoscopic puddles with locally $\kappa_{xy} = 3/2$ or $7/2$. Interactions between these puddles generate a coherent macroscopic state that exhibits a plateau with quantized $\kappa_{xy} = 5/2$. The non-Abelian quasiparticles characterizing this phase are distinct from those of the microscopic puddles and, by the same mechanism, could even emerge from a system comprised of microscopic Abelian puddles.

DOI: 10.1103/PhysRevLett.121.026801

Introduction.—The fractional quantum Hall (FQH) state at a Landau level filling factor $\nu = \frac{5}{2}$ was the first experimentally observed even-denominator state [1]. Soon after its discovery, pairing was suggested to play an important role in its formation [2,3], and several candidate paired states were proposed [2–7]. While the topological properties of these states have been thoroughly studied theoretically, their identification for the experimentally realized $\nu = \frac{5}{2}$ has proven difficult [8]. Numerically, exact diagonalization [9–15] and density matrix renormalization group (DMRG) studies [16,17] point towards the non-Abelian Pfaffian state, proposed by Moore and Read [2,3], and its particle-hole (PH) conjugate (the "anti-Pfaffian" [6,7]) as the most likely ground states.

Several experiments were carried out to differentiate between the candidate states. These include tunneling into the edge [18–21], noise measurements to probe the quasiparticle charge [22] (found to be $e^* = e/4$) and the existence of upstream neutral modes [23,24], interference [25], and—most recently—a measurement of the thermal Hall conductance [26]. The most unambiguous among these is the thermal Hall conductance, a topologically protected property whose quantized value differs between the candidate states. It was found experimentally to be $\kappa_{xy} = 5/2$ (in units of temperature times $\pi^2 k_B^2/3h$) [26]. Its half-integer value identifies the state as non-Abelian. It is, however, inconsistent with both Pfaffian and anti-Pfaffian, for which the values $7/2$ and $3/2$ are expected. Rather, $5/2$ is consistent with a non-Abelian state coined as the particle-hole Pfaffian (PH-Pfaffian), also known as T-Pfaffian [27–31] in the context of topological insulator surfaces. This phase first appeared in [7], and was recently reinterpreted in the context of Dirac composite fermions [32]. A prototypical wave function was proposed in [33], which also argued PH-Pfaffian to be most likely in light of earlier experiments. The (113) state [34] with $\kappa_{xy} = 2$ was also suggested, but deemed less likely.

Here we propose a resolution to the discrepancy between numerical predictions and experimental results. We focus on an ingredient that is undeniably present in experimental systems, but absent in numerical studies: quenched disorder. The possible stabilization of the PH-Pfaffian phase by disorder was suggested in [33]. We start with the observation that when PH symmetry is present, i.e., without Landau-level mixing, Pfaffian and anti-Pfaffian are degenerate at $\nu = \frac{5}{2}$. Spontaneous breaking of PH symmetry then determines which phase is realized. Deviating from this filling explicitly breaks PH symmetry and additionally introduces quasiparticles. In particular, if one phase prevails for $\nu > \frac{5}{2}$, its PH partner must be realized for $\nu < \frac{5}{2}$. With Landau-level mixing, PH is only an approximate symmetry at any filling and we expect the transition between the two phases to become shifted to $\nu^* \approx \frac{5}{2}$. For $\nu = \nu^* + \delta\nu$, we assume that $\delta\nu < 0$ favors Pfaffian while $\delta\nu > 0$ favors anti-Pfaffian.

In the presence of smooth density variations, we expect puddles of Pfaffian and anti-Pfaffian to form, whose size is much larger than the magnetic length, but smaller than the sample size. The disorder-induced puddle formation is strictly justified in the limit of zero-Landau level mixing where it follows from the statistical PH symmetry combined with a well-known argument by Imry and Ma [35] (see also [36]). We assume the same qualitative picture applies in the experimental systems without exact PH symmetry. The electronic Hall conductance is $\sigma_{xy} = \frac{5}{2}(e^2/h)$ independent of the relative areas occupied by

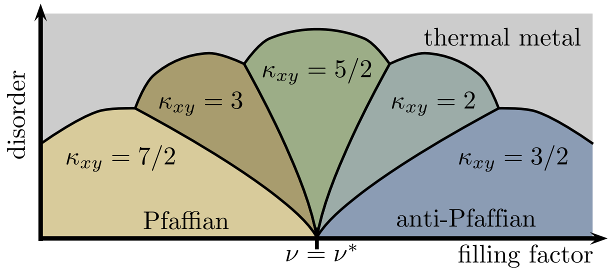

FIG. 1. Proposed phase diagram of the approximately half-filled first excited Landau level. Without disorder, the ground state is either Pfaffian or anti-Pfaffian, depending on whether the system is more particlelike ($\nu \lesssim \nu^*$) or holelike ($\nu \gtrsim \nu^*$) with an expected first-order transition at $\nu = \nu^* \approx \frac{5}{2}$. With disorder, this transition splits into four *continuous* phase transitions where the thermal Hall conductance changes by $\Delta\kappa_{xy} = 1/2$. For even stronger disorder, the system may enter a thermal metal, with nonquantized κ_{xy}. In contrast, the electrical Hall conductance $\sigma_{xy} = \frac{5}{2}(e^2/h)$ remains quantized across these transitions.

the two phases, since both have identical σ_{xy}. In contrast, κ_{xy} is determined by the predominant phase to be either $\kappa_{xy}^{\text{Pfaffian}} = 7/2$ or $\kappa_{xy}^{\text{Antipf.}} = 3/2$.

Our main result, summarized in Fig. 1, is that disorder splits the direct transition into a sequence of four transitions, each characterized by a change of $1/2$ in κ_{xy}. Between the transitions, κ_{xy} is sharply quantized to the values $3/2$, 2, $5/2$, 3, and $7/2$. For weak disorder the filling factor ν functions as a tuning parameter that drives the system across the four continuous quantum phase transitions. For fixed ν but increasing disorder strength, the system may transition into a thermal metal phase [37–41] with nonquantized κ_{xy}.

Pfaffianology.—Five candidate states for $\nu = \frac{5}{2}$ are relevant to the present work, each hosting multiple edges modes that interact among themselves. Beyond a length scale ℓ_{eq} the edge modes reach mutual equilibrium, at which point the state achieves a quantized thermal Hall conductance [42].

We will henceforth assume that all relevant length scales exceed ℓ_{eq}. (The values of ℓ_{eq}, as well as other microscopic length scales, are nonuniversal and difficult to estimate. Fortunately, these values are not crucial for our analysis; we will return to this point after explaining the phase diagram.)

All these states have two chiral ($\nu = 1$) electron edge modes of the two filled Landau levels, each contributing $\sigma_{xy} = 1$ and carrying central charge $c = 1$ and one copropagating ($\nu = \frac{1}{2}$) charge mode contributing $\sigma_{xy} = \frac{1}{2}$ and carrying $c = 1$; these modes contribute the required electrical Hall conductance, and a thermal Hall conductance of $\kappa_{xy} = 3$. The states differ by the number n_M of neutral Majorana modes (negative numbers indicate upstream modes), which determines the total thermal Hall conductance to be $\kappa_{xy} = 3 + n_M/2$. The values of n_M for all relevant phases are listed in Table I, and the edge modes

TABLE I. Summary of $\nu \approx 5/2$ phases relevant for our discussion. The number n_M of Majorana modes determines the thermal Hall conductance according to $\kappa_{xy} = 3 + n_M/2$.

QH phase	Pfaffian	$K = 8$	PH-Pfaffian	(113)	anti-Pfaffian
n_M	1	0	−1	−2	−3
κ_{xy}	7/2	3	5/2	2	3/2

of the non-Abelian phases are depicted in Fig. 2. Notice that only PH-Pfaffian ($n_M = -1$) is compatible with PH symmetry under which $\kappa_{xy} - \kappa_{xy}^{\nu=2} \to 1 - (\kappa_{xy} - \kappa_{xy}^{\nu=2})$, with $\kappa_{xy}^{\nu=2} = 2$. The corresponding edge states can be succinctly expressed in terms of their Lagrangian density $\mathcal{L}^{\chi}_{\text{vac.-QH}} = \mathcal{L}^{\chi}_{\text{c}} + \mathcal{L}^{\chi}_{\text{n}}[n_M]$ with

$$\mathcal{L}^{\chi}_{\text{c}} = \frac{1}{4\pi}\sum_{ij}[\chi K_{ij}\partial_t\phi_i\partial_x\phi_j - V_{ij}(\partial_x\phi_i)(\partial_x\phi_j)],$$

$$\mathcal{L}^{\chi}_{\text{n}}[n_M] = \sum_{i=1}^{|n_M|}\gamma_i[\partial_t - \chi\,\text{sgn}(n_M)v\partial_x]\gamma_i, \tag{1}$$

where ϕ are bosonic modes and γ are Majorana fermions; V is positive definite, $K = \text{diag}(1,1,2)$, $v > 0$, and $\chi = \pm 1$ (determining the chirality of the edge modes). An overview of the various edge modes appears in [43] (see also Fig. 6 in the extended material of [26]).

Disorder and network model.—We consider a disorder potential that is sufficiently weak and/or smooth that the results of numerical studies apply locally and favor the formation of Pfaffian or anti-Pfaffian puddles; see Fig. 3(a). The Pfaffian–anti-Pfaffian boundary is captured by [44]

$$\mathcal{L}^{\chi}_{\text{Pf.-APf.}} = \mathcal{L}^{\chi}_{\text{c}} + \mathcal{L}^{\chi}_{\text{n}}[1] + \mathcal{L}^{-\chi}_{\text{c}} + \mathcal{L}^{-\chi}_{\text{n}}[-3]. \tag{2}$$

Notice that when two quantum Hall states are separated by vacuum, fractional excitations cannot tunnel between the two. Still, the nonchiral Lagrangian of the charge modes $\mathcal{L}^{\chi}_{\text{c}} + \mathcal{L}^{-\chi}_{\text{c}}$ can be gapped by sufficiently strong tunneling of (pairs of) *electrons* across the Pfaffian–anti-Pfaffian boundary [44]. This "stitches together" the two quantum

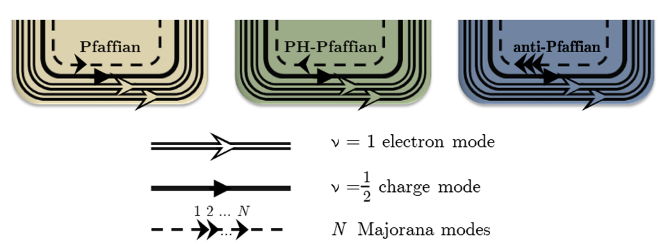

FIG. 2. Possible edge modes of three kinds of Pfaffian phases. All three phases have identical charge modes, but differ in the number and chirality of neutral Majorana modes (indicated by the number and direction of arrows) and consequently in their thermal Hall conductance.

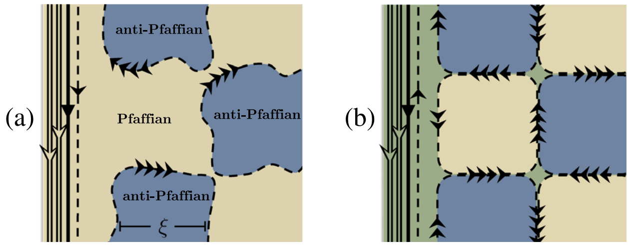

FIG. 3. Microscopic puddles and schematic network model. (a) Puddles of anti-Pfaffian (of size $\sim\xi$) embedded in a Pfaffian phase. Each Pfaffian–anti-Pfaffian boundary hosts four copropagating Majorana fermions. (b) Adding a topologically trivial pair of counterpropagating Majorana modes next to the sample boundary, followed by suitable hybridization of counterpropagating modes results in the PH-Pfaffian edge on top of a Chalker-Coddington model containing four Majorana fermions.

Hall states and permits fractional excitations to traverse. However, the neutral Majorana fermions copropagate and thus cannot be gapped out. Instead, the neutral terms in Eq. (2) combine into $\mathcal{L}_{\rm n}^{\chi}[4]$. Consequently, the Pfaffian–anti-Pfaffian boundary is described by four copropagating Majorana fermions, which have an $O(4)$ symmetry of rotating between them.

Our proposed model to describe a disordered $\nu = \frac{5}{2}$ system may be viewed as a checkerboard of alternating Pfaffian and anti-Pfaffian states as shown in Fig. 3(b), with random scattering at each vertex. (This is a generalization of the Chalker-Coddington network model, which has been highly successful in describing integer-quantum Hall plateau transitions [45]).

To gain intuition for the underlying physics, we first analyze the strongly anisotropic limit of the network model, which consists of (narrow) infinite strips that alternate between Pfaffian and anti-Pfaffian. A closely related model was studied in [44], focusing on the composite Fermi liquid, and in [31,46] to access the PH-Pfaffian phase in the topological-insulator-surface and quantum-Hall contexts, respectively. The low-energy properties of the model are encoded in $\mathcal{L}^{\rm anis} = \sum_y [\mathcal{L}_y^{\rm edge} + \mathcal{L}_y^{\rm tunnel}]$ with

$$\mathcal{L}_y^{\rm edge} = \vec{\gamma}_y \cdot [\partial_t - (-1)^y v \partial_x] \vec{\gamma}_{i,y},$$
$$\mathcal{L}_y^{\rm tunnel} = i\vec{\gamma}_y [M^{\rm unif} + (-1)^y M^{\rm stag} + M_y^{\rm rand}(x)] \vec{\gamma}_{y+1}, \quad (3)$$

where $\vec{\gamma}_y^T = (\gamma_{1,y}, \gamma_{2,y}, \gamma_{3,y}, \gamma_{4,y})$ and y enumerates Pfaffian–anti-Pfaffian interfaces. $\mathcal{L}_y^{\rm tunnel}$ is parametrized by three 4×4 matrices, describing uniform ($M^{\rm unif}$), staggered ($M^{\rm stag}$), and random ($M^{\rm rand}$) tunneling terms. The $O(4)$ symmetry of the nonrandom network makes $M^{\rm unif}$ proportional to the unit matrix. When $M^{\rm rand} = M^{\rm stag} = 0$, $\mathcal{L}^{\rm anis}$ features a discrete translation symmetry and is readily diagonalized in momentum space where one finds four gapless Majorana cones.

Perturbing these cones with a generic mass matrix $M^{\rm stag}$ opens an energy gap and the resulting phases can be classified according to the number of negative masses. A global anti-Pfaffian phase arises when all four cones are gapped by tunneling across Pfaffian strips. We adopt the convention that this corresponds to all Majorana masses being negative. When $M^{\rm stag}$ is $O(4)$ symmetric, there is a direct transition between Pfaffian and anti-Pfaffian. Without this symmetry individual masses can flip sign, each incrementing the total thermal Hall conductance by $\Delta\kappa_{xy} = 1/2$, thus realizing all phases in Table I.

We note that a clean uniform system differs from $\mathcal{L}^{\rm anis}$ without disorder. The transition between Pfaffian and anti-Pfaffian is expected to be first order in the former, but continuous in the latter. Consequently, intermediate phases with $\kappa_{xy} = 2$, $5/2$, and 3 are more readily accessible in the network model, by weakly perturbing the critical point. We expect this distinction to become insignificant in the presence of disorder where edge states between puddles necessitate closure of the energy gap, and the uniform and anisotropic models to exhibit the same universal behavior.

The key features of the anisotropic network model carry over to the isotropic case. Being free-fermion systems without charge conservation or any other symmetry, they fall into class D in the Altland-Zirnbauer classification [47,48]. In two dimensions, this class is characterized by a $\mathbb{Z}$ topological invariant n_M whose integer value corresponds to a quantized thermal Hall effect with $\kappa_{xy} = n_M/2$ [49,50]. Certain extra symmetries, such as the $O(4)$ symmetry in Eq. (3), can ensure transitions where κ_{xy} jumps by 2. However, this changes when disorder respects the protecting symmetry only *on average*. Random scattering completely mixes the four species of Majorana fermions and at large enough length scales, the system is effectively comprised of a single species of Majorana fermions. Without fine-tuning, such a system exhibits phase transitions across which the thermal Hall conductance changes by $\Delta\kappa_{xy} = 1/2$ (and not by larger $\Delta\kappa_{xy}$), leading to the proposed phase diagram shown in Fig. 1. The behavior around each of the four lines emanating from the transition point of the clean system was studied in [37–41]. We note that the disorder-induced localized phases in Fig. 1 are connectable to their clean analogs (113, PH-Pfaffian, and $K = 8$) without *delocalization* in the bulk. In contrast, the bulk energy gap is not protected in the disordered system and may close in this process, analogous to the case of the integer quantum Hall effect.

The emergent splitting of symmetry-protected phase transitions due to statistically symmetric disorder is familiar from other contexts. One example is the integer quantum Hall plateau transition between $\nu = 0$ and $\nu = 2$ of spin-degenerate electrons with spin-orbit scattering [51]. The introduction of spin-orbit scattering splits the single transition across which the Hall conductance changes by $\Delta\sigma_{xy} = 2(e^2/h)$ into two transitions, each with $\Delta\sigma_{xy} = (e^2/h)$. A second example is that of valley degenerate electrons in graphene, where random intervalley scattering splits the transition in a similar fashion [52].

In addition to the localized phases with quantized κ_{xy}, a thermal metal phase with nonquantized κ_{xy} can also arise [37–41]. There are thus two scenarios for a disordered system to transition between $\kappa_{xy} = 7/2$ (Pfaffian) and $\kappa_{xy} = 3/2$ (anti-Pfaffian): (i) The Majorana fermions in the bulk remain localized apart from sharp transitions with $\Delta\kappa_{xy} = 1/2$ each, or (ii) there is an intermediate delocalized phase and κ_{xy} varies continuously. Which case is realized depends on the type and strength of randomness, with an important role being played by vortex disorder. Ascertaining the fate of a particular system requires a detailed microscopic analysis that we do not attempt here. The experimental observation of a quantized thermal Hall conductance supports the first scenario over the second although it does not exclude the possibility of a metallic bulk with longitudinal thermal conductance $\kappa_{xx} \ll \kappa_{xy}$.

To corroborate our scenario, we numerically studied disorder-induced splitting of transitions between different topological phases in two superconducting model systems: a one-dimensional superconductor in class BDI and a two-dimensional superconductor in class D [49,50]. In the one-dimensional case, we studied four identical Kitaev-chains undergoing phase transitions between having and not having Majorana zero modes at their ends. The $O(4)$ symmetry inherent in this model ensures that there is a single transition, where the topological invariant changes by four. Upon introducing disorder that preserves this $O(4)$ symmetry only on average, we observe numerically that the transition splits into a sequence of four transitions (see also [53,54]).

A two-dimensional system where symmetry ensures a fourfold transition is a bilayer $p_x + ip_y$ superconductor transitioning into a $p_x - ip_y$ bilayer. This transition involves $\Delta n_M = 2$ in each layer, so that in total $\Delta n_M = 4$. The transition is protected by layer-interchange and spatial rotation symmetries. Again, we introduce statistically symmetric disorder and numerically observe splitting of the transition into a sequence of four transitions with $\Delta n_M = 1$ (see Supplemental Material [55] for details).

We note that our model is based on a fully coherent network. An *incoherent* mixture of puddles with two different values of the electrical Hall conductances exhibits the so-called semicircle law [57], which states that for approximately equal population of the phases, there is a large longitudinal conductance of the order of the difference between the two Hall conductances. We expect an analogous analysis for thermal transport in an incoherent mixture to lead to a similar law. This result is inconsistent with the observed plateau with $\kappa_{xy} = 5/2$. Yet, it is possible that it would describe samples of sizes larger than the coherence length.

We conclude our analysis of the network model by returning to the question of edge mode equilibration.

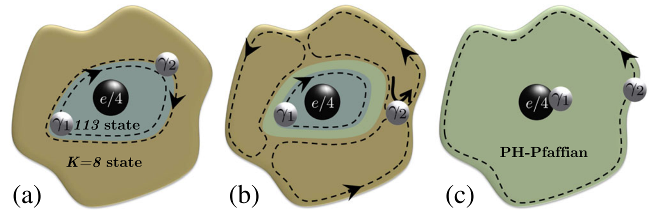

FIG. 4. Emergence of non-Abelian quasiparticles from puddles of Abelian phases. (a) In the Abelian “host” system, a fractional charge $e/4$ is associated with a pair of Majorana zero modes, which can hybridize with one another. (b) At the transition into the non-Abelian phase, only one of the two Majorana fermions (along with its zero mode) delocalizes throughout the system. (c) In the non-Abelian phase, the previously percolating Majorana fermion forms an edge state at the outer sample boundary, while still carrying the exact zero-energy mode.

Above, we always assumed that all edge modes are fully equilibrated at the length scale ξ. When $\xi < \ell_{\text{eq.}}$, the same model can still be used provided that puddles are interpreted as more coarse-grained objects: at a scale larger than $\ell_{\text{eq.}}$ and ξ, one may define the state of a “puddle” of this size to be whichever state is present in the majority.

Transmutation from Abelian to non-Abelian statistics.— A somewhat surprising outcome of our analysis is that puddles of two non-Abelian phases may form a macroscopic Abelian phase, and vice versa. On short length scales the quasiparticles reflect the state of the microscopic puddle where they reside. However, the topological properties of the macroscopic state can be inferred only from quasiparticles whose separation significantly exceeds both the puddle size and the localization length. Macroscopic degeneracy and non-Abelian braiding exist when each bulk quasiparticle carries a localized zero energy Majorana mode. In all the phases considered here, quasiparticles may be viewed as vortices in class-D superconductors, each harboring n_M mod 2 Majorana zero modes in their cores. The microscopic value of this number (determined by the puddle hosting the vortex) need not match the one of the macroscopic state. In that case, the difference must be made up by the effect of the vortex on the network of coupled Majorana modes.

We illustrate the mechanism behind this in Fig. 4, starting with puddles of two Abelian phases that form a macroscopic non-Abelian phase. Here, an $e/4$ excitation changes the boundary conditions of the edge states to create a *pair* of Majorana zero modes at the puddle boundary [cf. Fig. 4(a)]. This pair of zero modes is not protected and could thus hybridize and gap out. However, when the microscopic puddles of the two Abelian phases generate a macroscopic non-Abelian phase, one of the zero modes transfers to the sample boundary, leaving behind a single stable Majorana zero mode bound to the $e/4$ excitation [cf. Figs. 4(b) and 4(c)]. The complementary case of an Abelian phase arising from puddles of two non-Abelian phases can be understood analogously; see [55] for further details.

Conclusions and outlook.—We developed a model for a disordered system at $\nu \approx \frac{5}{2}$ built from mesoscopic puddles of Pfaffian and anti-Pfaffian. We showed, using both numerical and analytical arguments, that a plateau with $\kappa_{xy} = 5/2$ is stabilized at sufficiently long distances. Our theory predicts that for moderate disorder a series of phase transitions between κ_{xy} values of $7/2 \to 3 \to 5/2 \to 2 \to 3/2$ occurs with increasing filling factor ν. The properties of the quasiparticles at large distances, in particular their Abelian or non-Abelian statistics, are determined by the macroscopic phase, and not by the microscopic puddle in which they reside.

Experimental realization of the full series of transitions in κ_{xy} depends on the width of the $\sigma_{xy} = \frac{5}{2}(e^2/h)$ plateau, which is usually rather narrow. The widths of that plateau and of each of the phases in Fig. 1 increase with disorder (at least for weak disorder), but the scaling of their relative sizes is unknown to us. However, for a mesoscopic system, there may be an alternative route towards observing different quantization of the thermal Hall conductance. The splitting of plateaus only occurs beyond a crossover length scale, and smaller systems exhibit instead the thermal Hall conductance of Pfaffian or anti-Pfaffian. (A trivial example of this is a system containing only a single puddle.) A systematic study of the thermal Hall conductance as a function of sample size could thus be used to test our theory, as well as determine the crossover length scale.

Two upcoming works by D. Feldman and by B. Halperin *et al.* [36] study disorder-based mechanisms to explain the experimentally observed κ_{xy}. We thank them for sharing their insights on this topic. We would also like to thank Jason Alicea, Olexei Motrunich, Pavel Ostrovsky, Eran Sagi, Michael Zaletel, and especially Mitali Banerjee for fruitful discussions. This work was performed in part at the Aspen Center for Physics, which is supported by National Science Foundation Grant No. PHY-1607611 (D. F. M. and Y. O.) and partially supported by a grant from the Simons Foundation (D. F. M.); the Israel Science Foundation (D. F. M., Y. O., A. S., M. H.); the Minerva foundation with funding from the Federal German Ministry for Education and Research (D. F. M. and M. H.); the Binational Science Foundation (D. F. M. and Y. O.); the European Research Council under the European Community's Seventh Framework Program (FP7/2007-2013)/ERC Grant Agreement No. 339070 (M. H.) and Grant Agreement No. 340210 (Y. O. and A. S.); Microsoft Station Q (A. S.); the DFG (CRC/Transregio 183, EI 519/7-1) (Y. O. and A. S.); and the German Israeli Foundation, Grant No. I-1241-303.10/2014 (M. H.).

[1] R. Willett, J. P. Eisenstein, H. L. Störmer, D. C. Tsui, A. C. Gossard, and J. H. English, Phys. Rev. Lett. **59**, 1776 (1987).

[2] G. Moore and N. Read, Nucl. Phys. **B360**, 362 (1991).

[3] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).

[4] B. I. Halperin, Helv. Phys. Acta **56**, 75 (1983).

[5] M. Greiter, X. Wen, and F. Wilczek, Nucl. Phys. **B374**, 567 (1992).

[6] M. Levin, B. I. Halperin, and B. Rosenow, Phys. Rev. Lett. **99**, 236806 (2007).

[7] S.-S. Lee, S. Ryu, C. Nayak, and M. P. A. Fisher, Phys. Rev. Lett. **99**, 236807 (2007).

[8] A. Stern, Nature (London) **464**, 187 (2010).

[9] R. H. Morf, Phys. Rev. Lett. **80**, 1505 (1998).

[10] E. H. Rezayi and F. D. M. Haldane, Phys. Rev. Lett. **84**, 4685 (2000).

[11] M. R. Peterson, T. Jolicoeur, and S. Das Sarma, Phys. Rev. Lett. **101**, 016807 (2008).

[12] A. Wójs, C. Tőke, and J. K. Jain, Phys. Rev. Lett. **105**, 096802 (2010).

[13] M. Storni, R. H. Morf, and S. Das Sarma, Phys. Rev. Lett. **104**, 076803 (2010).

[14] E. H. Rezayi and S. H. Simon, Phys. Rev. Lett. **106**, 116801 (2011).

[15] K. Pakrouski, M. R. Peterson, T. Jolicoeur, V. W. Scarola, C. Nayak, and M. Troyer, Phys. Rev. X **5**, 021004 (2015).

[16] A. E. Feiguin, E. Rezayi, C. Nayak, and S. Das Sarma, Phys. Rev. Lett. **100**, 166803 (2008).

[17] A. E. Feiguin, E. Rezayi, K. Yang, C. Nayak, and S. Das Sarma, Phys. Rev. B **79**, 115322 (2009).

[18] I. P. Radu, J. B. Miller, C. M. Marcus, M. A. Kastner, L. N. Pfeiffer, and K. W. West, Science **320**, 899 (2008).

[19] X. Lin, C. Dillard, M. A. Kastner, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **85**, 165321 (2012).

[20] S. Baer, C. Rössler, T. Ihn, K. Ensslin, C. Reichl, and W. Wegscheider, Phys. Rev. B **90**, 075403 (2014).

[21] H. Fu, P. Wang, P. Shan, L. Xiong, L. N. Pfeiffer, K. West, M. A. Kastner, and X. Lin, Proc. Natl. Acad. Sci. U.S.A. **113**, 12386 (2016).

[22] M. Dolev, M. Heiblum, V. Umansky, A. Stern, and D. Mahalu, Nature (London) **452**, 829 (2008).

[23] A. Bid, N. Ofek, H. Inoue, M. Heiblum, C. L. Kane, V. Umansky, and D. Mahalu, Nature (London) **466**, 585 (2010).

[24] Y. Gross, M. Dolev, M. Heiblum, V. Umansky, and D. Mahalu, Phys. Rev. Lett. **108**, 226801 (2012).

[25] R. L. Willett, L. N. Pfeiffer, and K. W. West, Proc. Natl. Acad. Sci. U.S.A. **106**, 8853 (2009).

[26] M. Banerjee, M. Heiblum, V. Umansky, D. E. Feldman, Y. Oreg, and A. Stern, arXiv:1710.00492.

[27] X. Chen, L. Fidkowski, and A. Vishwanath, Phys. Rev. B **89**, 165132 (2014).

[28] P. Bonderson, C. Nayak, and X.-L. Qi, J. Stat. Mech. (2013) P09016.

[29] M. A. Metlitski, C. L. Kane, and M. P. A. Fisher, Phys. Rev. B **92**, 125111 (2015).

[30] C. Wang, A. C. Potter, and T. Senthil, Phys. Rev. B **88**, 115137 (2013).

[31] D. F. Mross, A. Essin, and J. Alicea, Phys. Rev. X **5**, 011011 (2015).

[32] D. T. Son, Phys. Rev. X **5**, 031027 (2015).

[33] P. T. Zucker and D. E. Feldman, Phys. Rev. Lett. **117**, 096802 (2016).

[34] G. Yang and D. E. Feldman, Phys. Rev. B **90**, 161306 (2014).
[35] Y. Imry and S.-k. Ma, Phys. Rev. Lett. **35**, 1399 (1975).
[36] C. Wang, A. Vishwanath, and B. I. Halperin, Phys. Rev. B **98**, 045112 (2018).
[37] S. Cho and M. P. A. Fisher, Phys. Rev. B **55**, 1025 (1997).
[38] T. Senthil and M. P. A. Fisher, Phys. Rev. B **61**, 9690 (2000).
[39] M. Bocquet, D. Serban, and M. Zirnbauer, Nucl. Phys. **B578**, 628 (2000).
[40] N. Read and A. W. W. Ludwig, Phys. Rev. B **63**, 024404 (2000).
[41] J. T. Chalker, N. Read, V. Kagalovsky, B. Horovitz, Y. Avishai, and A. W. W. Ludwig, Phys. Rev. B **65**, 012506 (2001).
[42] C. L. Kane, M. P. A. Fisher, and J. Polchinski, Phys. Rev. Lett. **72**, 4129 (1994).
[43] G. Yang and D. E. Feldman, Phys. Rev. B **88**, 085317 (2013).
[44] M. Barkeshli, M. Mulligan, and M. P. A. Fisher, Phys. Rev. B **92**, 165125 (2015).
[45] J. T. Chalker and P. D. Coddington, J. Phys. C **21**, 2665 (1988).
[46] X. Wan and K. Yang, Phys. Rev. B **93**, 201303 (2016).
[47] M. R. Zirnbauer, J. Math. Phys. **37**, 4986 (1996).
[48] A. Altland and M. R. Zirnbauer, Phys. Rev. B **55**, 1142 (1997).
[49] S. Ryu, A. P. Schnyder, A. Furusaki, and A. W. W. Ludwig, New J. Phys. **12**, 065010 (2010).
[50] A. Kitaev, AIP Conf. Proc. **1134**, 22 (2009).
[51] D. K. K. Lee and J. T. Chalker, Phys. Rev. Lett. **72**, 1510 (1994).
[52] P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, Phys. Rev. B **77**, 195430 (2008).
[53] O. Motrunich, K. Damle, and D. A. Huse, Phys. Rev. B **63**, 224204 (2001).
[54] M.-T. Rieder, P. W. Brouwer, and I. Adagideli, Phys. Rev. B **88**, 060509 (2013).
[55] Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.121.026801, which contains numerical results showing disorder-induced splitting of phase transitions in models of 1D and 2D superconductors; explains the emergence of non-Abelian quasiparticles from Abelian puddles; and includes [56].
[56] Y.-F. Zhang, Y.-Y. Yang, Y. Ju, L. Sheng, R. Shen, D.-N. Sheng, and D.-Y. Xing, Chin. Phys. B **22**, 117312 (2013).
[57] A. M. Dykhne and I. M. Ruzin, Phys. Rev. B **50**, 2369 (1994).