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Genuine Quantum Nonlocality in the Triangle Network

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Quantum networks allow in principle for completely novel forms of quantum correlations. In particular, quantum nonlocality can be demonstrated here without the need of having various input settings, but only by considering the joint statistics of fixed local measurement outputs. However, previous examples of this intriguing phenomenon all appear to stem directly from the usual form of quantum nonlocality, namely via the violation of a standard Bell inequality. Here we present novel examples of "quantum nonlocality without inputs," which we believe represent a new form of quantum nonlocality, genuine to networks. Our simplest examples, for the triangle network, involve both entangled states and joint entangled measurements. A generalization to any odd-cycle network is also presented.

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Introduction.—Bell's theorem is arguably among the most important results in the foundations of quantum theory [1]. It also had a major influence on the development of quantum information science [2], and led recently to the so-called device-independent paradigm [3–6].

In his seminal work, Bell demonstrated that two distant observers, performing local measurements on a shared entangled state, can establish strong correlations which cannot be explained in any physical theory satisfying a natural principle of locality. These nonlocal quantum correlations can be demonstrated experimentally using Bell inequalities. Recently, long-awaited loophole-free tests of quantum nonlocality were finally reported, providing the basis for the implementation of device-independent quantum information protocols [7–9].

An interesting direction is to understand quantum nonlocality in scenarios involving more than two observers. The standard approach to this problem (referred to as multipartite Bell nonlocality) considers three (or more) distant observers sharing an entangled state distributed by a common source, and leads to interesting new effects; see, e.g., Ref. [10] for a review. This represents the simplest generalization of quantum nonlocality to the multipartite case, and most of the concepts and tools developed for bipartite nonlocality can generally be directly extended here.

Recently, a completely different approach to multipartite nonlocality was proposed [11–13], focusing on quantum networks. Here, distant observers share entanglement distributed by several sources which are assumed to be independent from each other. By performing joint entangled measurements (such as the well-known Bell state measurement used in quantum teleportation [14]), observers may correlate distant quantum systems and establish strong correlations across the entire network. Typically, each source connects here only a strict subset of the observers. It turns out that this situation is fundamentally different from standard multipartite nonlocality, and allows for radically novel phenomena. As regards correlations, it is now possible to witness quantum nonlocality in experiments where all the observers perform a fixed measurement; i.e., they receive no input [12,13, 15–19]. This effect of quantum nonlocality without inputs is remarkable, and radically departs from previous forms of quantum nonlocality.

So far, however, all known examples of quantum nonlocality without inputs can be traced back to standard Bell inequality violation. This naturally leads to the question of whether completely novel forms of quantum nonlocality, genuine to the network configuration, could arise. Here we address this question, by presenting an instance of quantum nonlocality in the triangle network, which we argue is fundamentally different from previously known forms of quantum nonlocality. In particular, our construction crucially relies on the combination of shared entangled states and joint entangled measurements performed by the observers. We present several generalizations of our main result, in particular to any cycle network featuring an odd number of parties. We conclude with a discussion and comment on the main open questions.

Scenario and main result.—We consider the so-called triangle quantum network sketched in Fig. 1. It features three observers (Alice, Bob, and Charlie). Every pair of observers is connected by a (bipartite) source, providing a shared physical system (represented, e.g., by a classical variable or by a quantum state). Importantly, the three

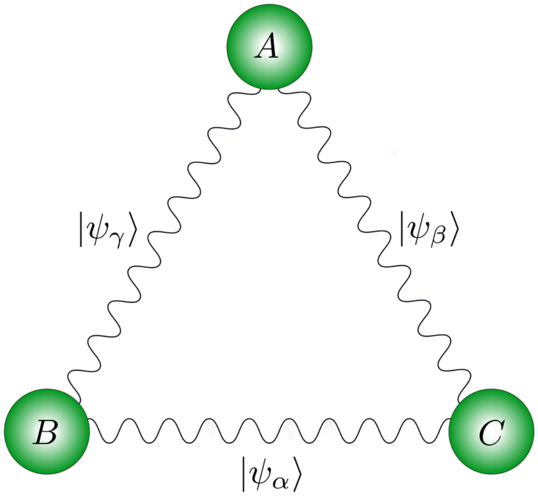

FIG. 1. The triangle network features three observers (green circles), connected by three independent bipartite sources. Here, the sources distribute the quantum states ψ_α, ψ_β, and ψ_γ.

sources are assumed to be independent of each other. Hence, the three observers share no common (i.e., tripartite) piece of information. Based on the received physical resources, each observer provides an output (a, b, and c, respectively). Note that the observers receive no input in this setting, contrary to standard Bell nonlocality tests. The statistics of the experiment are thus given by the joint probability distribution $P(a,b,c)$.

Characterizing the set of distributions $P(a,b,c)$ that can be obtained from physical resources (in particular, classical or quantum) is a highly nontrivial problem. The main difficulty stems from the assumption that the sources are independent. This makes the set of possible distributions $P(a,b,c)$ nonconvex, and standard methods used in Bell nonlocality are thus completely unadapted to this problem. Strong bounds on the limits of classical correlations are thus still missing, which in turn renders the discussion of quantum nonlocality in the triangle network challenging.

Here we follow a different approach in order to present instances of quantum nonlocality in the triangle network. Specifically, we first construct explicitly a family of quantum distributions $P_Q(a,b,c)$, using both entangled quantum states (distributed by each of the three sources), and entangled joint measurements performed by each observer. Then we show that these quantum distributions cannot be reproduced by any "trilocal" model, i.e., a local model "a la Bell" where all three sources are assumed to be independent from each other. Formally, we prove that

$$P_Q(a,b,c) \neq \int d\alpha \int d\beta \int d\gamma P_A(a|\beta,\gamma) P_B(b|\gamma,\alpha) P_C(c|\alpha,\beta), \quad (1)$$

where $\alpha \in X$, $\beta \in Y$, and $\gamma \in Z$ represent the three local variables distributed by each source and $P_A(a|\beta,\gamma)$, $P_B(b|\gamma,\alpha)$, $P_C(c|\alpha,\beta)$ represent arbitrary deterministic response functions for Alice, Bob, and Charlie. Our proof does not rely on the violation of some Bell-type inequality, but is based on a logical contradiction. More precisely, we first identify a certain number of necessary properties that any trilocal model should have in order to reproduce $P_Q(a,b,c)$, and then show that these properties cannot all be satisfied at the same time.

Let us now construct explicitly our quantum distributions $P_Q(a,b,c)$. Each source produces the same pure maximally entangled state of two qubits,

$$|\psi_\gamma\rangle_{A_\gamma B_\gamma} = |\psi_\alpha\rangle_{B_\alpha C_\alpha} = |\psi_\beta\rangle_{C_\beta A_\beta} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

Note that each party receives two independent qubit subsystems; for instance, Alice receives subsystems A_β and A_γ. Next, each party performs a projective quantum measurement in the same basis. In the following, we use the basis (a set depending on one real parameter u) given by

$$\begin{aligned} |\uparrow\rangle &= |01\rangle, & |\chi_0\rangle &= u|00\rangle + v|11\rangle, \\ |\downarrow\rangle &= |10\rangle, & |\chi_1\rangle &= v|00\rangle - u|11\rangle, \end{aligned} \quad (2)$$

with $u^2 + v^2 = 1$ and $0 < v < u < 1$. For Alice, we label it $\{|\phi_a\rangle_{A_\beta A_\gamma}\}$ for $\phi_a \in \{\uparrow, \downarrow, \chi_0, \chi_1\}$ and adopt similar notations for Bob and Charlie. Remark that only two out of the four states in that basis are entangled. The statistics of the experiment are given by

$$P_Q(a,b,c) = |\langle\phi_a|\langle\phi_b|\langle\phi_c||\psi_\gamma\rangle|\psi_\alpha\rangle|\psi_\beta\rangle|^2,$$

where we did not specify the Hilbert spaces supporting the states. Note that when evaluating $P_Q(a,b,c)$, one should be attentive to which Hilbert space supports each state and measurements. We now state the main result of this Letter:

Theorem I: The quantum distribution $P_Q(a,b,c)$ cannot be reproduced by any classical trilocal model [in the sense of Eq. (1)] when $u_{\max}^2 < u^2 < 1$, where $u_{\max}^2 = \{[-3 + (9 + 6\sqrt{2})^{2/3}]/[2(9 + 6\sqrt{3})^{1/3}]\} \approx 0.785$.

We now sketch the proof; all details are given in Appendix A of the Supplemental Material [20]. The main idea is that the quantum distribution $P_Q(a,b,c)$ features a certain number of specific constraints. Indeed, one has that

$$P_Q(a = \uparrow, b = \uparrow) = P_Q(a = \downarrow, b = \downarrow) = 0. \quad (3)$$

Symmetric relations are obtained by permuting the parties. Also, the number of parties that have an output in $\chi = \{\chi_0, \chi_1\}$ must be odd. Moreover, introducing the notation $u_0 = -v_1 = u$ and $v_0 = u_1 = v$ (such that $|\chi_t\rangle = u_t|00\rangle + v_t|11\rangle$) we have that

$$P_Q(\chi_i,\uparrow,\downarrow) = \frac{1}{8}u_i^2, P_Q(\chi_i,\downarrow,\uparrow) = \frac{1}{8}v_i^2, \qquad (4)$$

$$P_Q(\chi_i,\chi_j,\chi_k) = \frac{1}{8}(u_i u_j u_k + v_i v_j v_k)^2 \qquad (5)$$

and similar relations by permuting the parties. These four properties are essentially all we need. Indeed, for some specific choice of the measurement parameter u, no trilocal model can be compatible with all these four constraints at once. We prove by contradiction, assuming a trilocal model, in two successive steps where we identify conditions that this trilocal model reproducing $P_Q(a,b,c)$ should fulfill, to finally arrive at a contradiction.

Step 1.—Here we consider the coarse graining of the output set $\{\uparrow,\downarrow,\chi = \{\chi_0,\chi_1\}\}$. We show that the sources sets can be partitioned in two subsets of equal weight $X = X_0 \coprod X_1$, $Y = Y_0 \coprod Y_1$, $Z = Z_0 \coprod Z_1$ such that upon receiving β and γ, Alice outputs (i) $a = \uparrow$ if she receives $\beta \in Y_0$ and $\gamma \in Z_1$, (ii) $a = \downarrow$ if she receives $\beta \in Y_1$ and $\gamma \in Z_0$, (iii) $a = \chi$ otherwise (similarly for Bob, Charlie, see Fig. 2).

Proof.—See Appendix B of the Supplemental Material [20], it relies on Eqs. (3)–(5). ∎

Step 2.—Let us introduce the following probability distribution

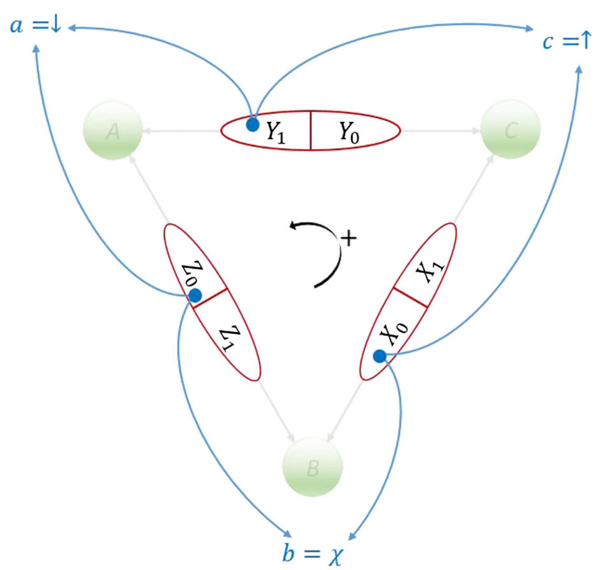

FIG. 2. Step 1 shows that any trilocal model compatible with the quantum distribution $P_Q(a,b,c)$ must have a specific structure. Specifically, for each source, the classical variable set can be divided into two equal weight [i.e., $P(\alpha \in X_0) = \ldots = 1/2$] disjoint subsets containing all output information on the coarse grained distribution where $\chi = \{\chi_0,\chi_1\}$ group together two outputs. For instance, when $\alpha \in X_0$, $\beta \in Y_1$, and $\gamma \in Z_0$, the outputs must be $a = \downarrow$ for Alice, $b = \chi$ for Bob, $c = \uparrow$ for Charlie. Note that the ordering is important.

$$q(i,j,k,t) \coloneqq 4p[a = \chi_i, b = \chi_j, c = \chi_k, (\alpha,\beta,\gamma) \in (X_t, Y_t, Z_t)]. \qquad (6)$$

The following marginal distributions of $q(i,j,k,t)$ satisfy

$$q(i,j,k) = \sum_t q(i,j,k,t) = \frac{1}{2}(u_i u_j u_k + v_i v_j v_k)^2, \qquad (7)$$

$$q(i,t=0) = \sum_{j,k} q(i,j,k,t=0) = \frac{1}{2}u_i^2, \qquad (8)$$

and similar constraints on $q(i,t=1)$, $q(j,t)$, and $q(k,t)$.

Proof.—From Step 1, one can see that all parties output χ iff $(\alpha,\beta,\gamma) \in (X_t,Y_t,Z_t)$ with $t=0$ or $t=1$. This ensures that $q(i,j,k,t)$ is properly normalized. Equation (7) is straightforward from Eq. (5). Equation (8) can be deduced from step 1 and the fact that Alice's output must be independent of α (see Appendix A of the Supplemental Material [20] for a more detailed proof). ∎

At this point we arrive at a contradiction. Indeed, if a trilocal model existed, one should be able to define a distribution $q(i,j,k,t)$ that is compatible with all its marginals, in particular those marginals discussed above. However, this is not possible for all values of the parameter u (which quantifies the degree of entanglement of measurement χ_0,χ_1), specifically when $0,785 \approx u_{\max}^2 < u^2 < 1$. This concludes the proof.

A natural question is whether the distribution P_Q is trilocal when $u^2 \le u_{\max}^2$. In Appendix D of the Supplemental Material [20], we show that this is the case, by constructing an explicit trilocal model for $u^2 = u_{\max}^2$ (up to machine precision). We conjecture that P_Q remains trilocal up to $u^2 < u_{\max}^2$. Note that this can be proven for the case $u^2 = 1/2$. Here the trilocal model is obtained from step 1, with χ replaced by a uniformly random choice between χ_0 and χ_1.

Before entering a more general discussion about the implications of Theorem 1 and some natural open questions, we now briefly present several generalizations of the result.

Generalizations.—The first extension considers the same scenario as in Theorem 1, with the difference that all sources now produce the same general entangled two-qubit pure states $\lambda_0|00\rangle + \lambda_1|11\rangle$, where $\lambda_0^2 + \lambda_1^2 = 1$. We consider the same measurements (2). In this case, Theorem 1 can be extended, with the condition that $u_{\max}(\lambda_0) \lesssim u < 1$ (see Appendix A of the Supplemental Material [20] for details). Interestingly, the lower bound $u_{\max}(\lambda_0)$ takes its lowest value for nonmaximally entangled states ($\lambda_0 = \sqrt{2/3}$). In this case, we find $u_{\max}(\lambda_0) = \sqrt{2/3}$, implying that the projective joint measurement must feature nonmaximally entangled states.

A second generalization considers the triangle network with higher dimensional quantum systems. Specifically, all three sources now produce a maximally entangled two-qutrit state, i.e., $|\phi_3\rangle = (|00\rangle + |11\rangle + |22\rangle)/\sqrt{3}$. Each party performs the same joint entangled projective measurement, with nine outcomes, labeled by $a \in \{\tilde{0}, \tilde{1}, \tilde{2}, \chi_0^\uparrow, \chi_1^\uparrow, \chi_2^\uparrow, \chi_0^\downarrow, \chi_1^\downarrow, \chi_2^\downarrow\}$ for Alice. The projectors are on the states

$$|\tilde{0}\rangle \coloneqq |00\rangle, \qquad |\tilde{1}\rangle \coloneqq |11\rangle, \qquad |\tilde{2}\rangle \coloneqq |22\rangle,$$
$$|\chi_i^\uparrow\rangle = \eta_i^{01}|01\rangle + \eta_i^{02}|02\rangle + \eta_i^{12}|12\rangle,$$
$$|\chi_i^\downarrow\rangle = \eta_i^{10}|10\rangle + \eta_i^{20}|20\rangle + \eta_i^{21}|21\rangle,$$

where the coefficients $\{\eta_i^{01}, \eta_i^{02}, \eta_i^{12}\}$ and $\{\eta_i^{10}, \eta_i^{20}, \eta_i^{21}\}$ are real and chosen such that the nine vectors form an orthonormal basis. Similarly to Theorem 1, one can show that for an appropriate choice of these parameters, the resulting quantum distribution is incompatible with any trilocal model (see Appendix B of the Supplemental Material [20]).

A third generalization explores networks beyond the triangle. Specifically, we prove a generalization of Theorem 1 for any N-cycle network, with N being odd. Here all N sources produce a maximally entangled two-qubit state, and all parties perform the same joint measurement, as in Eq. (2). We show that for any N, the quantum distribution is incompatible with any N-local model [i.e., a straightforward generalization from Eq. (1)] when the measurement parameter u goes asymptotically to 1. Our approach cannot be directly adapted to even cycles.

Discussion.—We presented novel examples of quantum nonlocality without inputs, mainly for the triangle network. We believe that these examples represent a form of quantum nonlocality that is genuine to the network configuration, in the sense that it is not a consequence of standard forms of Bell nonlocality. These examples fundamentally differ from the one presented by Fritz in Ref. [13] (as well as related examples in Ref. [17]), relying on the violation of a standard bipartite Bell inequality. Let us first briefly review it.

Fritz's example can be viewed as a standard Bell test, embedded in the triangle network. Consider that Alice and Bob share a two-qubit Bell state, with the goal of violating the CHSH Bell inequality. Testing the CHSH inequality requires of course local inputs for both Alice and Bob. Although the triangle network features no explicit inputs, here effective inputs are provided by the two additional sources: the source connecting Alice and Charlie (Bob and Charlie) provides a shared uniformly random bit, which is used as Alice's (Bob's) input for the CHSH test. All parties output the "input bits" he receives. The correspondence between these outputs ensures that Alice's (Bob's) output only depends on the source she (he) shares with Charlie. Finally, Alice and Bob both additionally output the output of their local measurement performed on the shared Bell state. If this quantum distribution could be reproduced by a trilocal model, it would follow that local correlations can violate the CHSH inequality, which is impossible.

Let us comment on some significant differences between Fritz's construction and our example of Theorem 1. First, our construction has a high level of symmetry (all sources distribute the same entangled state and all measurements are the same) with only four outputs per party. In particular, it involves an entangled state for each source, whereas the example of Fritz requires entanglement for only one source (it can be symmetrized, but at the cost of adding new outputs). Lastly, our example appears to rely on the use of joint measurements with entangled eigenstates, while Fritz's model uses only separable measurements. Hence Fritz's construction could be obtained from PR boxes [21]. As the equivalent of joint measurements does not exist for PR boxes [22,23], we believe that our example cannot be obtained from PR boxes.

Note that all the above arguments are only based on qualitative and intuitive arguments. Still, we believe that the judicious combination of entangled states and joint entangled measurements is the key for a new form of quantum nonlocality. Proving that the distribution $P_Q(a,b,c)$ can only be obtained using three entangled sources and/or joint entangled measurements would represent significant progress. An idea would be to use the notion of "self-testing" [24], for instance, by proving that all shared quantum states must be two-qubit Bell states and/or that all local measurements must feature specific entangled eigenstates [25,26].

Another important aspect of our construction that must be discussed is noise tolerance. As such, Theorem 1 clearly applies only to the exact quantum distribution $P_Q(a,b,c)$, i.e., in the noiseless case. The trilocal set being topologically closed, it is clear that $P_Q(a,b,c)$ must have a certain (possibly very weak) robustness to noise: when adding a sufficiently small amount of local noise to $P_Q(a,b,c)$, one should still obtain a quantum distribution that is incompatible with any trilocal model. A promising method would be to consider the qutrit example, the proof of which involves the Finner inequality that allows, in principle, for the presence of noise. However, we did not succeed in obtaining reasonable noise tolerance of our result so far. Other methods could also help, such as the "inflation" technique [15] or the finite cardinality of the classical variables [27,28]. This could provide a nonlinear Bell inequality violated by our example.

The possibility of generating randomness from quantum nonlocality without inputs is a further interesting question. In particular, it seems very likely that our quantum distribution $P_Q(a,b,c)$ contains some level of intrinsic randomness. It would be interesting to see how this randomness could be quantified in a device-independent manner (still assuming independence of the sources).

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