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Direct Measurement of a Nonlocal Entangled Quantum State

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Entanglement and the wave function description are two of the core concepts that make quantum mechanics such a unique theory. A method to directly measure the wave function, using weak values, was demonstrated by Lundeen *et al.* [Nature **474**, 188 (2011)]. However, it is not applicable to a scenario of two disjoint systems, where nonlocal entanglement can be a crucial element, since that requires obtaining weak values of nonlocal observables. Here, for the first time, we propose a method to directly measure a nonlocal wave function of a bipartite system, using modular values. The method is experimentally implemented for a photon pair in a hyperentangled state, i.e., entangled both in polarization and momentum degrees of freedom.

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A wave function description plays an important role in quantum theory, while its objective reality gives rise to a century-long debate [1–6]. Using the technique of weak measurement that enables one to obtain the weak value of a pre- and postselected quantum system [7,8], a method of directly measuring the complex wave function of single photons was experimentally demonstrated recently [9]. The technique was subsequently extended to discrete two- [10] or higher-dimension [11–13] quantum systems and even mixed states [14,15]. The method was found to have deep connections to the phase space distributions [16,17] and sequential measurements [18–20]. It was developed with strong measurements [21–23] and further applied to measure matter waves [24,25]. In none of these tasks [26,27] could the measured wave function be related to two disjoint systems and thus could not represent nonlocal entanglement. Here, for the first time, we show a direct measurement of a wave function with nonlocal entanglement. We achieve this by using modular values [28], which enable one to obtain the weak value of a (nonlocal) product of observables.

A general wave function $|\Psi\rangle$ can be written using a basis $|n\rangle$ as $|\Psi\rangle = \sum_n \Psi_n |n\rangle$, where Ψ_n are complex amplitudes. A projective measurement of $|n\rangle$ would yield only $|\Psi_n|^2$, and not any phase information, so it was a surprise when Lundeen *et al.* [9] showed that using weak values one can directly measure both the real and imaginary parts of Ψ_n. A weak value of an observable O, on a system that is prepared in a state $|\psi\rangle$ and postselected to a state $|\phi\rangle$, is given by $O_w = \langle\phi|O|\psi\rangle/\langle\phi|\psi\rangle$. It is a complex quantity, in contrast to the expectation value or any of the eigenvalues, which are always real. The weak value of a projection operator $P_n = |n\rangle\langle n|$ with a postselection on uniform superposition $|\phi\rangle \propto \sum_n |n\rangle$ yields the complex amplitudes $\psi_n \propto (P_n)_w$. The standard technique to obtain a weak value, known as weak measurement, is via an interaction described by the evolution operator $U_I = e^{-igOp}$, where $g \ll 1$ is a dimensionless coupling constant and p is an operator on a meter. After the interaction and postselection on the system, the expectation value $\langle p\rangle$ and $\langle x\rangle$, with x being an operator conjugate to p, will change according to $\delta p = 2g(\langle p^2\rangle - \langle p\rangle^2)\Im[O_w]$ and $\delta x = g\Re[O_w]$, respectively [29].

Consider the case that our system is composed of two subsystems, A and B, placed at different locations. The complete Hilbert space is a tensor product of the two local Hilbert spaces $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. One can define local bases for each subsystem $|j\rangle_A \in \mathcal{H}_A$, $|l\rangle_B \in \mathcal{H}_B$ such that the basis for the complete system is a tensor product of local states $|n\rangle = |j,l\rangle = |j\rangle_A|l\rangle_B$, with amplitudes $\Psi_{j,l}$ given by the weak value of projection operators $P_{j,l} = P_j^A P_l^B$. However, the interaction $U_I = e^{-ig(P_{j,l})p}$, which is needed in the standard scheme to obtain $(P_{j,l})_w$, is not physical, regardless of what p might be, since it requires a nonlocal Hamiltonian $H \propto P_j^A P_l^B$. Such a Hamiltonian implies an instantaneous interaction between distant locations. Thus, the method described above cannot be applied to this case and it seems that one cannot directly measure a nonlocal wave function. This implies a major flaw in the implication of the direct measurement technique. Apart from the ongoing discussion regarding the efficiency of the method, this flaw is related to the fundamental aspects of the idea and can render it useless for the most interesting cases. Without the locality restriction, one can adopt a realistic description and thus the wave function is redundant. Any potential application of the method would also be highly limited due to the pivotal role of entanglement in many quantum protocols. Here we show that this is not the case, by introducing a new method and experimentally demonstrating it, where the weak values are replaced, or rather augmented, by modular values [28].

Modular values were introduced as an explanation of an experiment demonstrating the Hardy paradox [30,31], which involved the weak value of a product, and as a method to obtain weak values using strong measurement. Later on, it extended theoretically in several ways [32] and was also implemented experimentally [33].

Since the (nonlocal) observable we are interested in, $P_{l,j}$, is a product of two (local) observables, the problem boils down to obtaining the weak value of a product of observables. This task cannot be done using the standard weak measurement technique, where the meter evolves according to a Hamiltonian in which the observable on the system is replaced by its weak value $O \to O_w$. A way to achieve this task was initially suggested in [34] and later realized [35]. Their method relies on a second-order term, while still requiring the interaction to be weak. A product of N observables would be obtained from the Nth order, so the scalability of this method poses significant challenges. The method we use, based on modular values, has the additional benefit of allowing one to obtain weak values using strong measurement. In [28], it was shown that a qubit meter, interacting via an observable O on a pre- and postselected system, evolves according to the modular value, given by $O_m = \langle\phi|e^{-igO}|\psi\rangle/\langle\phi|\psi\rangle$, where g is a coupling constant of arbitrary size. When the relevant observable is a projection, the modular value has a close connection to the weak value $(P_n)_m = 1 + s(P_n)_w$, with $s = e^{-ig} - 1$. We set $s = -2$, which corresponds to a standard experimental setting. In the case of two commuting projectors, we have

$$\Psi_{j,l} \propto (P_j^A P_l^B)_w = s^{-2}[(P_j^A + P_l^B)_m - (P_j^A)_m - (P_l^B)_m + 1], \quad (1)$$

where for any single projector on a subsystem there is an implicit tensor product with an identity operator on the other subsystem. The first expression in Eq. (1) is implied by the original method [9], while the second expression comes directly from the definition of the modular value. While qubit meters are typically used to obtain modular values, the projection observable P_j can pertain to a continuous variable such as the position or velocity of a particle [36,37], with the indices j, l in Eq. (1) denoting the continuous property. Thus, the problem of measuring directly a nonlocal wave function is mapped to directly measuring the modular values of observables such as $P_j^A + P_l^B$, which we now show how to accomplish, using an entangled meter.

Since the meter should interact with both subsystems, it should also consists of two parts, even though in principle one can also have a single meter and move it to each location of the subsystems. After each part interacts with one subsystem and the system is postselected, the modular value can be extracted from the final state of the meter by tomography [28]. The tomography in the last step can be replaced by a more direct method by setting the meter in an initial state $|\Psi_I^m\rangle = (|\uparrow\downarrow\rangle + \epsilon|\downarrow\uparrow\rangle)/\sqrt{1+\epsilon^2}$, where the first (second) arrow refers to the part of the meter interacting with subsystem $A(B)$ and $\epsilon \ll 1$. Then, after an interaction $U_I = e^{-ig(P_j^A P_\downarrow^A + P_l^B P_\uparrow^B)}$ with $P_\downarrow^A$ ($P_\uparrow^B$) a projection on the part $A(B)$ of the meter to the state $|\downarrow\rangle$ ($|\uparrow\rangle$), the probabilities $\mathcal{P}_1$ and $\mathcal{P}_2$ of finding the meter in the states $|1\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ and $|2\rangle = (|\uparrow\downarrow\rangle + i|\downarrow\uparrow\rangle)/\sqrt{2}$, respectively, are given by

$$\mathcal{P}_1 = \frac{1}{2} + \epsilon\Re(P_j^A + P_l^B)_m + O(\epsilon^2), \quad (2)$$

$$\mathcal{P}_2 = \frac{1}{2} + \epsilon\Im(P_j^A + P_l^B)_m + O(\epsilon^2). \quad (3)$$

Thus, to the first order in ϵ, the readout in certain detectors will be given by the real and imaginary part of the relevant modular value. To obtain $(P_j^A)_m$ and $(P_l^B)_m$, one can set the meter initially in a product state and look at the probability of finding states, similar to $|1\rangle$ and $|2\rangle$, for each part of the meter separately. Alternatively, one can replace in the interaction U_I, a projector on one part with an identity operator (for more details see the Supplemental Material [38]).

Note that $\epsilon \ll 1$ does not imply the procedure is a weak measurement in the sense that the interaction parameter g does not have to be small. One can set ϵ to be large as well and still reconstruct weak values from Eq. (1), which means using strong measurements. Indeed, an alternative to the direct measurement technique, based on strong measurement, was theoretically proposed [21,22] and experimentally demonstrated [23]. That technique could also be interpreted using modular values. In our scheme, we choose $\epsilon \ll 1$, so using Eqs. (2) and (3), the complex amplitudes of a wave function with nonlocal entanglement, appear naturally in the measurement results.

The general scheme of our method is shown in Fig. 1 and the experimental setup is shown in Fig. 2. More details are given in the Supplemental Material [38]. In the experiment, we have measured the wave function of the polarization of two photons using their paths' degree of freedom as a meter, such that the state $|\uparrow\rangle$ ($|\downarrow\rangle$) for a part of the meter implies that the photon went through the right (left) arm. We used hyperentangled photon pairs such that both the polarization and path states are entangled between the

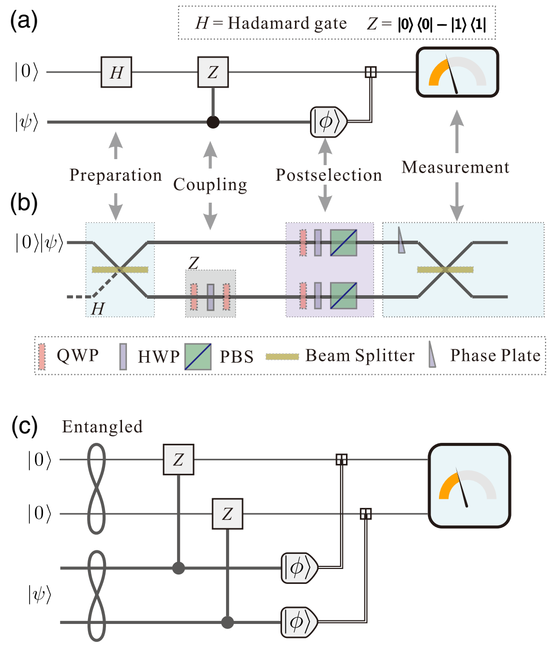

FIG. 1. Measurement of a pre- and postselected system, using qubit meters. (a) A logic circuit diagram shows a single-qubit meter prepared in a state $|\uparrow_x\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and a single system prepared in a state $|\psi\rangle$. Then, a controlled gate modifies the state of the meter, depending on the state of the system and, finally, if the system is found in a state $|\phi\rangle$, the meter is readout. (b) Experimental scheme to realize the circuit in (a) using the path of a photon as a meter and its polarization as the system [half-wave plate (HWP), quarter-wave plate (QWP), polarized beam splitter (PBS)]. (c) The logic circuit diagram for the case of a bipartite system (two spatiality separated qubits) is composed from similar components to the single-qubit case shown in (a) and (b). The main difference is that the system can be initially entangled and thus the meter might have to be entangled as well.

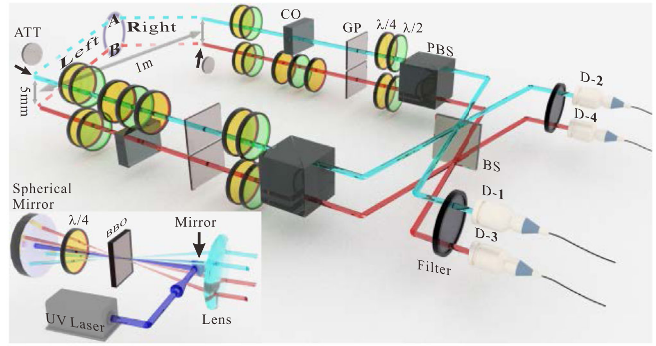

FIG. 2. Experimental setup. (Inset) A β-BaB$_2$O$_4$ (BBO) crystal pumped by a vertically polarized ultraviolet laser. A spherical mirror reflects the photon pairs, which are emitted due to a degenerate spontaneous parametric down-conversion, and the pump beam to a second pass through the crystal, after passing a QWP (or $\lambda/4$) with its optical axis orienting at 45°. Choosing four points in the entanglement ring yields a photon pair entangled both in path and polarization, which enters the main setup shown in the main panel. The up path (green) stands for the first photon (labeled A), while the bottom path (red) represents the second one (labeled B); they are vertically separated around 5 mm. For each photon, they have two spatially separated path modes (left and right with around one meter from each other) and two polarization modes (horizontal and vertical); the former is taken as the meter and the latter as the system. For preparing the initial meter state, we use two neutral optical attenuators (ATTs) to precisely reduce the relative intensity of relevant modes. Half- and quarter-wave plates are used to manipulate the polarization of each path independently and glass compensators (COs) are used to offset the phase difference. The two paths of each photon are combined in one unpolarized beam splitter (BS). Glass plates (GPs) are used to introduce a controllable phase so different path states can be measured. The PBS postselects the final polarization state. After spectrum filters, the photons are collected by four single-mode fibers and guided to four avalanche single-photon detectors (D-1,2,3,4).

photons, while there is no entanglement, initially, between the polarization and path [39]. The probabilities in Eqs. (2) and (3) are obtained by Franson interference. Since the initial state of the meter is entangled, we detect product states $|\tilde{1}\rangle = (|\uparrow\rangle + |\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle)/2$ and $|\tilde{2}\rangle = (|\uparrow\rangle + i|\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle)/2$ to obtain $\mathcal{P}_1$ and $\mathcal{P}_2$, respectively.

We start by demonstrating our method for a single component of the state. We prepare the system in a maximally entangled state $1/\sqrt{2}(|HH\rangle + e^{i\theta}|VV\rangle)$ with an adjustable phase $\theta \in \{-\pi, \pi\}$ and postselect the system to $(|H\rangle + |V\rangle)(|H\rangle + |V\rangle)/2$, where $|H\rangle$ and $|V\rangle$ denote the horizontal and vertical polarization, respectively. The relevant modular values and the probability amplitude are shown in Fig. 3. The method yields the expected values for $|\theta| \lesssim \pi/2$. For $|\theta| \simeq \pi$, where the initial state is nearly orthogonal to the postselection, the modular value diverges, as does the weak value, while the probability amplitude does not. The relation in Eq. (1) still holds since the proportionality constant in the first expression vanishes accordingly. However, higher orders in Eqs. (2) and (3) cannot be neglected anymore. This problem can be solved by choosing a different postselection, for example $(|H\rangle + |V\rangle)(|H\rangle - |V\rangle)/2$. Another solution would be to make ϵ smaller if the interferometer is sufficiently ideal with high interference visibility.

In Fig. 4 we present the complete probability amplitudes that are measured using our method for a number of states. In our case, the three modular values used to measure the single component, as shown in Fig. 3, are enough to obtain all the components, since $P_H = I - P_V$, with I as the identity matrix, and $(cI + O)_m = s^c(O)_m$, for any observable O and number c. A system of dimension n would have $n-1$ independent projectors, so for a general bipartite system composed of subsystems of dimensions m and n, one needs $(m-1) + (n-1)$ single-system modular values and $(m-1) \times (n-1)$ two-system combinations, i.e., products of projectors on the separate systems. Adding

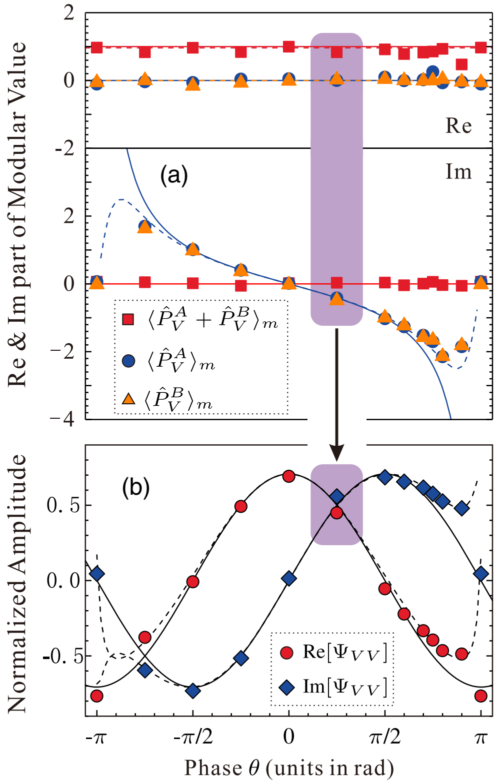

FIG. 3. Demonstration of the method for the state $1/\sqrt{2}(|HH\rangle + e^{i\theta}|VV\rangle)$. (a) Modular values (upper, for real part, and lower, for imaginary part): To avoid clutter, we only show the corresponding values for the projector P_V^A, P_V^B, and $P_V^A + P_V^B$. (b) The normalized probability amplitude for the corresponding component Ψ_{VV} of the state. While all the real parts of modular values in (a) exhibit constant value, the oscillations in both the real and imaginary part of Ψ_{VV} emerge when performing the normalization to the whole state. In both (a) and (b), solid lines are values derived from a perfectly prepared state, dashed lines are predicted results taking into account the higher order of ϵ ($= 0.2$ in our case), and markers are experimental results. The error bars (calculated through Monte Carlo simulations considering the counting noise) are smaller than the point size.

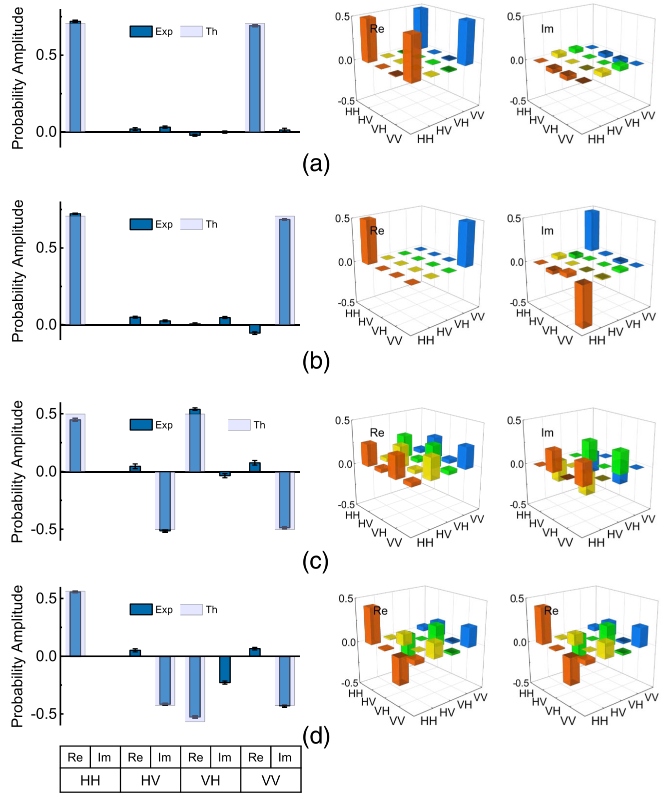

FIG. 4. Experimentally measured probability amplitudes. The upper part of each panel shows the directly measured probability amplitudes (Exp). The theoretical values (Th) refer to the state in the case of an ideal preparation, which in each panel is given by (a) $|\Psi\rangle=(|HH\rangle+|VV\rangle)/\sqrt{2}$, (b) $|\Psi\rangle=(|HH\rangle+i|VV\rangle)/\sqrt{2}$, (c) $|\Psi\rangle=(|H\rangle+|V\rangle)(|H\rangle-i|V\rangle)/2$, and (d) $[0.8|HH\rangle-0.6i|HV\rangle-0.8|VH\rangle-0.6i|VV\rangle]/\sqrt{2}$. The lower part of each panel shows the reconstructed density matrix (colored bars) for each state, obtained by standard tomography. Errors are estimated through Monte Carlo simulations considering the counting noise.

these numbers and multiplying by 2, for real and imaginary parts, yields exactly the number of independent parameters $2m \times n - 2$ in a state of dimension $n \times m$. For comparison, we show in Fig. 4 the reconstructed density matrices obtained by standard tomography, requiring 16 different measurements for each state. Our Letter is limited to pure states, and reconstructing mixed states will necessarily be more resource intensive.

In addition to the main result, our method illuminates an aspect of the direct measurement technique, which is sometimes overlooked or misunderstood. Essentially, it demonstrates that the ability to obtain the probability amplitude is due to weak values rather than weak measurements. It is the postselection that enables us to achieve the task and not necessarily a small interaction strength. While this distinction could be inferred directly from the theoretical derivation, supporting it by experimental results can help clarify this issue, as well as the discussion regarding the efficiency of various techniques, especially with regards to the supposed benefits of small interactions.

More importantly, extending the method of direct measurement to scenarios having nonlocal entanglement will allow using it to study, theoretically and experimentally, many ideas, such as steering, quantum discord, entanglement entropy, etc. The extended method could be incorporated into quantum protocols for which the original method was not applicable due to the locality constraint on the measured wave function. Since it is not yet clear where one would use the original method, and how, we can only speculate regarding concrete applications of our method. It is possible that some future technology would require obtaining the wave function of a pure state, which is nonlocally entangled. The context can be information transfer, executing a distributed computational task, cryptography protocols, etc. Then, one might find that our method is required for an efficient implementation of such technology.

In conclusion, we have experimentally demonstrated a direct measurement of nonlocal wave functions for the first time. The task is achieved by using modular values of a sum of observables that yield the weak values of nonlocal observables. The method sheds new light on the previous technique and extends it to be applicable for an important scenario: the existence of nonlocal entanglement. We anticipate that our results can inspire the direct measurement of multipartite states in some other quantum systems where weak values are accessible at present, such as atoms [40], neutrons [41], superconductors [42], etc. Introducing sequential measurements [18–20] into our method and developing it for directly measuring mixed states should be worthy of further studies. The simplicity of the theoretical derivation and the demonstrated feasibility of the experimental technique can make the new method a powerful tool for studying the nature of quantum mechanics and harnessing it.

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