We study the jamming phase diagram of sheared granular material using a novel Couette shear setup with a multiring bottom. The setup uses small basal friction forces to apply a volume-conserving linear shear with no shear band to a granular system composed of frictional photoelastic discs. The setup can generate arbitrarily large shear strain due to its circular geometry, and the shear direction can be reversed, allowing us to measure a feature that distinguishes shear-jammed from fragile states. We report systematic measurements of the stress, strain, and contact network structure at phase boundaries that have been difficult to access by traditional experimental techniques, including the yield stress curve and the jamming curve close to $\phi_{SJ} \approx 0.75$, the smallest packing fraction supporting a shear-jammed state. We observe fragile states created under large shear strain over a range of $\phi < \phi_{SJ}$. We also find a transition in the character of the quasistatic steady flow centered around $\phi_{SJ}$ on the yield curve as a function of packing fraction. Near $\phi_{SJ}$, the average contact number, fabric anisotropy, and nonrattler fraction all show a change of slope. Above $\phi_f \approx 0.7$ the steady flow shows measurable deviations from the basal linear shear profile, and above $\phi_h \approx 0.78$ the flow is localized in a shear band.

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Experimental measurements of the phase boundaries in the jamming phase diagram are challenging because it is hard to create SJ states without the formation of a shear band and the associated heterogeneities in the packing fraction $\phi$ and strain field \cite{10,25–29}. In 2013, Ren et al. \cite{10} developed a multislit, simple shear setup that avoids shear banding, which revealed a distinction between $F$ and SJ states \cite{21,23}. However, their multislit setup had a strain limit (~60\%) \cite{10}, and thus could not access the yield stress curve or the SJ states near $\phi_{\text{SJ}}$, where $\gamma_{\text{SJ}}$ keeps growing as $\phi \to \phi_{\text{SJ}}$ \cite{11,15,16}.

In this Letter, we solve this challenge using a multiring Couette shear setup, which applies a linear shear strain field using basal friction forces to drive the system until it becomes shear jammed. This form of driving may be thought of as a physical implementation of the algorithm used in certain athermal, quasistatic simulations \cite{20,30}.

With our apparatus, we can also keep shearing the jammed system using boundary racks to measure the yield stress curve. By shearing a layer of photoelastic disks, we for the first time experimentally map out the phase boundaries in the jamming phase diagram close to $\phi_{\text{SJ}}$, including the yield stress curve and the jamming curve. We find that fragile states exist below $\phi_{\text{SJ}}$ that were not included in the traditional phase diagram \cite{1}. Moreover, we find two transitions on the yield stress curve: (i) above $\phi_{\text{F}} \approx 0.7$, the steady states no longer deform linearly under shear, and (ii) above $\phi_{\phi} \approx 0.78$ their deformation field becomes localized. We relate those transitions to the contact network structures.

Experiments.—The experiments are carried out with a novel multiring Couette shear setup shown in Fig. 1(b), which quasi- and linearly shears a 2D granular medium composed of bidisperse photoelastic discs with friction coefficient 0.9 and diameters 1.59 and 1.27 cm (denoted as $d$) \cite{31}. The ratio of the numbers of big and small particles is 1/3. Particles have reflective paint on their surfaces to enable reflective photoelasticimetry \cite{34–37}. The total number of particles is varied from 1447 to 2101, which corresponds to 0.56 $< \phi < 0.82$. The Couette setup consists of 21 independently controlled concentric rings. The 1.2 cm wide rings rotate collectively, providing weak frictional forces to the particles sitting on them. Although essential to perform the linear shear, the magnitude of the basal friction is ~8 times smaller than the typical contact forces measured in the SJ states on the jamming curve [Fig. 1(a)]. Particles are constrained radially by outer and inner toothed boundaries of radius $r_{\text{out}} = 35.5$ cm and $r_{\text{in}} = 8.7$ cm. The outer boundary rotates with the rings and the inner boundary is fixed.

For each experiment, a stress-free random configuration is prepared. The quasistatic linear shear is then applied in a stepwise manner. For each step, the ring at radial position $r$ rotates through an arc length $d(r) = yr$. The function $d(r)$ sets the “basal profile” and $\gamma$ is called the “shear strain” by analogy with traditional simple shear \cite{10}. We note that $\gamma$ is not the physical shear strain, i.e., the off-diagonal element of the strain tensor, $\varepsilon_{\gamma\theta} = 0, d(r) - d(r)/(r + r_m) = \gamma r_m/(r + r_m)$ \cite{38}. During a rotation step, in which $\delta \gamma = 0.6\%$, the shear rate is $\gamma \approx 10^{-3}$ s$^{-1}$. After each step, the rings stop for 10 s to let the system reach a static state. As plotted in Fig. 1(c), for a dilute system, the azimuthal particle displacements $u_{\phi}$ per step follow $d(r)$, and the radial displacements $u_r$ fluctuate around zero. No shear band is observed. We apply large forward strains to measure the yield stress curve, and the strain direction is then reversed to distinguish fragile and shear-jammed states.

The system is sequentially lit from the top by circular polarized green light, and from the side by ultraviolet (uv) light \cite{31}. Between two consecutive shear steps, after reaching a static state, the system is imaged (Canon EOS 70D, $5472 \times 3648$ px$^2$) through a circular polarizer with uv and polarized lights. Ultraviolet images [Fig. 1(e)] give particle positions. The polarized images [Fig. 1(d)] give stress and contact information. We measure the pressure $P$, defined as the force moment tensor \cite{1,10}, using the averaged squared intensity gradient \cite{9,10,36,37,39,40} of the polarized image \cite{31}. A sheared system must develop a nonzero $P$ to resist finite shear stress $\tau$. We also measure the nonrattler contact number $Z_{\text{nr}}$, defined as the mean contact number among stressed grains \cite{31,36,41}, the nonrattler fraction $f_{\text{nr}}$, defined as the number fraction of stressed grains, and the fabric anisotropy $\rho$, defined as the ratio between the difference and the sum of the eigenvalues of the fabric tensor \cite{31}. (See Ref. \cite{36} for a detailed description of the contact detection algorithm.)

Results.—Figures 2(a) and 2(b) show pressure $P$ and nonrattler contact number $Z_{\text{nr}}$ versus shear strain $\gamma$, for typical runs with different $\phi$. For a given $\phi$, after a transient growth regime, both $Z_{\text{nr}}$ and $P$ fluctuate around constant values that define the yield stress curve. We refer the associated stress as the “steady state” stress. We find that $Z_{\text{nr}}$ can be fitted to

$$Q = Q_{\text{r}} + ce^{-\gamma/C\gamma},$$

where $Q$ can be $Z_{\text{nr}}$, $f_{\text{nr}}$, or $1 - \rho$, and $Q_{\text{r}}$, $c$, and $C\gamma$ are fit parameters. An example fit for $Z_{\text{nr}}(\gamma)$ with $\phi = 0.76$ is plotted in Fig. 2(b). We find that the steady regime has been reached at $\gamma_{\text{r}} \approx 3\gamma_c$ for all state variables, where $\gamma_c$ is obtained from the fits for $Z_{\text{nr}}$. Figure 2(c) shows $\gamma_{\text{r}}(\phi)$, where a linear fit $\gamma_{\text{r}} \propto (\phi - \phi_0)$ for $\phi > 0.72$ gives $\phi_0 = 0.84 \pm 0.02$, close to the frictionless isotropic jamming density \cite{42}. The slope is $-1545 \pm 427$ ($\%$).

We identify a system as shear jammed if under reverse shear the pressure never drops below the noise threshold $P_{\text{noise}} = 0.3$ N/m \cite{31}, which indicates that the system resists the reversed stress rather than simply allowing a reversion to a stress-free (unjammed) state. Figure 2(d) gives an example of $P$ during a shear cycle for a system.
also follows Eq. (2). In this fit, we take our data. Below \( \phi \) in the reverse shear process. As shown in Fig. 3(a), we find \( \gamma_{\text{SJ}} \) value near the noise level.

\[
\gamma_{\text{SJ}}(\phi) = \gamma_b \left[ \ln \left( \frac{\phi_J - \phi_{\text{SJ}}}{\phi_{\text{SR}} - \phi} \right) \right]^\alpha,
\]

with \( \phi_{\text{SJ}} = 0.781 \). Figure 2(e) plots the dependence of the minimum pressure \( P_{\text{min}} \) during reverse shear on the maximum forward shear strain \( \gamma_{\text{max}} \), from which we extract the minimum strain, \( \gamma_{\text{SJ}} \), required to create a SJ state. We find no SJ state for \( \phi = 0.74 \) even when \( \gamma_{\text{max}} \gg \gamma_{\text{st}} \) [31]. For \( \phi = 0.75 \), we find \( \gamma_{\text{SJ}} \approx \gamma_{\text{st}} \). The minimum packing fraction that supports shear jamming must lie between these two values: \( \phi_{\text{SJ}} = 0.745 \pm 0.005 \). Figure 3(a) plots the relation between \( \gamma_{\text{SJ}} \) and \( \phi \), which can be fitted using a form suggested in Ref. [15],

\[
\gamma_{\text{SJ}}(\phi) = \gamma_b \left[ \ln \left( \frac{\phi_J - \phi_{\text{SJ}}}{\phi_{\text{SR}} - \phi} \right) \right]^\alpha,
\]

where \( \phi_{\text{SJ}} = 0.745 \) is preset and the fit parameters are \( \alpha = 0.68 \pm 0.11 \), \( \gamma_b = 64 \pm 6 \% \), and \( \phi_J = 0.820 \pm 0.005 \).

In this work, fragile (\( F \)) states refer to states with nonzero pressure (\( P > P_{\text{noise}} \)) and have \( P_{\text{min}} < P_{\text{noise}} \) at some point in the reverse shear process. As shown in Fig. 3(a), we find \( \gamma_F \), the minimum strain required to create a fragile state, also follows Eq. (2). In this fit, we take \( \phi_J = 0.82 \) from the previous fit, and we determine \( \phi_F \), the minimum packing fraction for fragile states, from the fit, obtaining \( \phi_F = 0.706 \pm 0.003 \) along with \( \gamma_b = 19 \pm 2 \% \) and \( \alpha = 0.86 \pm 0.12 \). We also note, however, that the divergence predicted by Eq. (2) near \( \phi_{\text{SJ}} \) and \( \phi_F \) is not clearly seen in our data. Below \( \phi_F \), the steady state pressure falls to a plateau value near the noise level.

\[
\frac{1}{P_{\text{SR}}(\phi)} = \frac{1}{P_{\text{st}}} + a \phi + b \phi^2,
\]

with \( \phi_{\text{SR}} = 0.50 \) and \( \phi_{\text{st}} = 0.781 \). Figure 3(b) shows the relation between \( \gamma_{\text{SR}} \) and \( \phi \), which can be fitted using a form suggested in Ref. [15].

\[
\gamma_{\text{SR}}(\phi) = \gamma_{\text{SR}}^0 + a (\phi - \phi_{\text{SR}}^0)^2.
\]

where \( \phi_{\text{SR}}^0 = 0.50 \) and \( \gamma_{\text{SR}}^0 = 0.781 \). Figure 3(c) plots the relation between \( \gamma_{\text{SR}} \) and \( \phi \), which can be fitted using a form suggested in Ref. [15].

\[
\gamma_{\text{SR}}(\phi) = \gamma_{\text{SR}}^0 + a (\phi - \phi_{\text{SR}}^0)^2.
\]
medium in a way that maintains a linear shear strain profile until the system becomes jammed, allowing us to probe the jamming transition close to $\phi_{SJ}$. The setup subsequently shears the jammed system using the boundary racks, allowing a study of the yield stress curve for a wide range of packing fractions. Finally, reversing the direction of the drive allows us to distinguish SJ from F states.

We systematically measured the phase boundaries in the jamming phase diagram, including close to $\phi_{SJ}$, leading to the following key observations. (i) In our system $\phi_{SJ} \approx 0.75$, whose value may depend on the friction coefficient $\mu$, polydispersity, and particle shape, though we expect the qualitative features of the jamming phase diagram to be the same. (ii) The SJ strain $\gamma_{SJ}$ is well fit by a stretched logarithmic function of $\phi$. The measured exponent $\alpha = 0.68 \pm 0.11$ is in quantitative agreement with the exponent $\alpha = 1/1.37 \approx 0.73$ measured from the simulation of the sheared 3D frictionless soft spheres [15]. The same form, but with $\alpha = 1$, has also been observed in experiments on shear-thickening suspensions [5]. (iii) We observe fragile states below $\phi_{SJ}$, which are not included in the traditional phase diagram [1]. In our system, small basal friction forces and particle deformability may be crucial for stabilizing the fragile force network. (iv) On the yield stress curve, for increasing packing fraction, we find that $P_{st}$ has an inflection point at $\phi_{SJ}$ and that $Z_{nr, st}$, $P_{st}$, and $f_{nr, st}$ all show a change of slope near $\phi_{SJ}$, suggesting a physical transition in the nature of the steady states.

We also find that the quasistatic steady flow field changes from the nonlocalized basal profile for systems with $\phi < \phi_c \approx 0.7$ to a localized shear band for $\phi > \phi_b \approx 0.78$, where $\phi_c < \phi_{SJ} < \phi_b$. The coexistence of a solid and fluid phase in slowly sheared dense granular matter has been reported in many systems [25,27,28,43–46]. In this Letter, we characterize the contact network associated with the different quasistatic steady flow regimes. When $\phi = \phi_b$, the steady states have $\rho_{st} \approx 0.05$ and $f_{nr, st} \approx 1$, showing a nearly isotropic, fully percolated contact network. Notably, $\phi_b \approx \phi^*_b$ with $\mu \approx 0.9$, where $\phi^*_b$ is the isotropic jamming packing fraction with friction coefficient $\mu$ [47]. We also note that $Z_{nr, st}(\phi_{SJ}) \approx 3.4$, similar to the mean contact number observed when a strong force network percolates in both principal directions in biaxial experiments [1], and $Z_{nr, st}(\phi_{F}) \approx 3.9$, close to the isostatic value for ideal frictionless disks [42,48].

The results suggest several directions for further study. First, our shear device can generate other basal profiles [35] to study how shear jamming affects the granular rheology for shear fields found in real world applications. Second, the setup can create a controlled shear band, providing a new technique to study the generation and evolution of shear bands in dense granular flow.

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* yiqiu.zhao@duke.edu
† tzhenghu@gmail.com
‡ socolar@phy.duke.edu
Deceased, July 2018.

[31] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.123.158001 for (1) more details of the setup, (2) a stress strain curve of a single particle, (3) the pressure calibration, (4) the fabric tensor calculation, (5) raw data for reverse shear tests, and (6) the choice of noise level, which includes Refs. [32,33].