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Lighting the Dark: Evolution of the Postinflationary Universe

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In simple inflationary cosmological scenarios, the near-exponential growth can be followed by a long period in which the Universe is dominated by the oscillating inflaton condensate. The condensate is initially almost homogeneous, but perturbations grow gravitationally, eventually fragmenting the condensate if it is not disrupted more quickly by resonance or prompt reheating. We show that the gravitational fragmentation of the condensate is well-described by the Schrödinger-Poisson equations and use numerical solutions to show that large overdensities form quickly after the onset of nonlinearity. This is the first exploration of this phase of nonlinear dynamics in the very early Universe, which can affect the detailed form of the inflationary power spectrum and the dark matter fraction when the dark sector is directly coupled to the inflaton.

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In simple inflationary scenarios, the Universe grows at least 10^{60} times larger between the Planck scale and the present day [1–5]. Roughly speaking, the inflationary phase accounts for 30 of these 60 factors of 10, smoothing away preinflationary remnants, and laying down the perturbations that seed galaxy formation and the anisotropies in the microwave sky. In the trillionth of a second after inflation, the Universe grows by another 15 factors of 10, at which point, typical interactions take place at Large Hadron Collider energies. The remaining growth occurs during the 13.8 billion years elapsing between that point and the present day. Roughly half (logarithmically) of the post-inflationary growth of the Universe occurs in an eye blink and at energies beyond the reach of current experiments. This epoch is the primordial dark age [6]: its unknown dynamics are critical to understanding both ultra-high-energy particle physics and the infant Universe.

The Universe must thermalize before neutrino freeze-out and can do so via several mechanisms. The inflaton condensate can fragment into its own quanta via self-resonance, and particles coupled to the inflaton can be resonantly produced off the condensate [7–9], leading to prompt thermalization [10] or a possible oscillon-dominated epoch [11–15]. The full dynamics of this phase must typically be simulated via three dimensional Klein-Gordon solvers in a rigid, expanding spacetime [16–18]. Without resonance, particles are generated by slower, perturbative processes [19–21]. This perturbative mechanism takes places during a lengthy period of expansion in which the Universe is dominated by a coherent, nearly homogeneous condensate [22], which eventually fragments via the gravitational growth of perturbations [23,24]. However, this process has not been studied beyond the breakdown of perturbation theory. Most treatments of resonance ignore local gravitational effects while fully relativistic solvers [25–27] need to resolve the “fast” dynamics of the oscillating field, which is prohibitively expensive.

We show that the growing perturbations in the inflaton condensate are well described by the Schrödinger-Poisson equations [28–32] and can be simulated with tools [32–38] similar to those used with ultralight dark matter [ULDM] [39–42] or self-interacting axions [43]. We simulate the fragmentation of a coherently oscillating inflaton condensate into localized, gravitationally confined overdensities, a key step toward the understanding of the primordial dark age in this class of models. This is key to fixing the detailed inflationary perturbation spectra [44–47] and for making quantitative predictions in models where the dark matter is coupled to the inflaton (e.g., [48–52]) or consists of remnants of the inflaton itself [53–56].

Scenario.—The inflaton, ϕ, obeys the equation of motion

$$\nabla_\mu \nabla^\mu \phi - V'(\phi) = 0, \tag{1}$$

and the metric obeys the Einstein field equations. In the homogeneous limit, these reduce to

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \quad H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right), \tag{2}$$

where $M_{\rm Pl} = \sqrt{1/8\pi G}$ is the reduced Planck mass, $H = \dot{a}/a$, a is the usual scale factor, and V is the potential. Current constraints require $V(\phi)$ to be subquadratic at large field values [57]. Nonquadratic potentials can induce self-resonance, fragmenting the inflaton condensate before the gravitational growth of perturbations becomes significant [15]. Consequently, we assume that the inflaton oscillates in a purely quadratic minimum, or

$$V(\phi) = \frac{1}{2} m^2 \phi^2. \tag{3}$$

and that the potential grows more slowly than ϕ^2 only at larger values of the field. The postinflationary evolution of the homogeneous field is then

$$\phi = \sqrt{\frac{8}{3}} \frac{M_p}{m} \frac{1}{t} \sin(mt), \tag{4}$$

up to arbitrary constants, while $a(t) \sim t^{2/3}$ and the comoving horizon grows as $t^{1/3}$, or $a^{1/2}$. This is equivalent to a matter dominated period of expansion.

For this analysis we need only the initial perturbation spectrum, and we assume that modes that never left the horizon have vanishing amplitude, while the dimensionless metric perturbations are scale invariant outside the horizon. That is, the power spectrum of the comoving curvature perturbation is

$$\mathcal{P}_{\mathcal{R}}(k) = \begin{cases} A & \text{for } k \lesssim k_{\text{horizon}} \\ 0 & \text{for } k \gtrsim k_{\text{horizon}} \end{cases}. \tag{5}$$

The value of A is sensitive to the form of $V(\phi)$ as inflation ends, which we have not specified, so A is a free parameter.

Perturbations only grow inside the horizon. Modes just outside the horizon at the end of inflation reenter first and, thus, undergo the most growth, with their amplitudes increasing linearly with the expansion of the Universe during the linear regime. Thus, we expect the first structures to form on comoving scales slightly larger than the Hubble radius at the end of inflation. Generically, superhorizon modes will have some scale dependence, but unless this is extreme, it will be swamped by extra growth undergone by shorter modes that spend longer inside the horizon.

The Schrödinger-Poisson regime.—The homogeneous oscillation timescale in Eq. (4) is $1/m$. For our analysis, we set $m = \sqrt{3} H_{\text{end}}$, where H_{end} is the Hubble parameter at the end of inflation. The horizon scales as $1/H \sim a^{3/2}$, while the oscillation time, $1/m$, stays constant. Consequently, a few e-folds after inflation ends, $1/H \gg 1/m$, and numerically evolving the full Klein-Gordon dynamics for several Hubble times with realistic parameter values is computationally infeasible. Moreover, at the onset of nonlinearity, perturbations of interest are safely subhorizon, bulk motions are nonrelativistic, and occupation numbers are high. These are precisely the circumstances in which the (Newtonian) Schrödinger-Poisson formalism is applicable: matter is represented by the nonrelativistic wave function ψ and the gravitational potential Φ is found by solving the Poisson equation.

The Schrödinger-Poisson equations are derived by applying the ansatz [31]

$$\phi = \frac{1}{m a^{3/2}} (\psi e^{-imt} + \psi^* e^{imt}), \tag{6}$$

to Eq. (1) and the Einstein field equations, where ψ is a complex variable varying over both time and space. This factors out the homogeneous oscillations in Eq. (4). One then separates out the $e^{\pm imt}$ components and makes the approximation $m \gg |\dot{\psi}/\psi|$. This resembles the WKB approximation, in that dynamics on the timescale of the condensate oscillations are assumed to be ignorable. The result is

$$i\dot{\psi} = -\frac{1}{2ma^2} \nabla^2 \psi + m\psi\Phi, \tag{7}$$

$$\nabla^2 \Phi = \frac{4\pi G}{a} (|\psi|^2 - \langle |\psi|^2 \rangle), \tag{8}$$

and the matter density is given by $|\psi|^2$.

The Schrödinger-Poisson equations also describe structure formation with ULDM [39–42,58]. To explore their early Universe analog, we have generalized PyUltraLight [36] to an expanding background with appropriate initial conditions, along with a dynamical time step. (This code will be described in a forthcoming publication.) Simulations have been run at resolutions of up to 512^3. The spatial resolution must be sufficient to resolve the change in the phase of ψ at adjacent grid points. One can understand this through the fluid approximation where the velocity is proportional to the gradient of the phase. If the phase changes by more than π between any neighboring grid points, the fluid velocity is not well defined, and the code halts when the phase of any two neighboring grid points changes by more than $\pi/2$. The simulation volume is a fixed comoving region with periodic boundary conditions.

In the linear regime, the density perturbations are

$$\psi = \sqrt{\rho_c} \sqrt{1 + \delta} e^{iS}, \tag{9}$$

where ρ_c is the critical density at the end of inflation, and the Fourier representations of δ and S are [32]

$$\delta_k(a) = \tilde{A}_k R_k(a), \tag{10}$$

$$S_k(a) = -\frac{d\delta_k}{dx}, \tag{11}$$

with

$$R_k(a) = \left(\frac{3}{x^2} - 1\right) \cos(x) + \frac{3}{x} \sin(x), \tag{12}$$

$$x = \frac{k^2}{m\sqrt{H_{\text{end}}^2 a}}. \tag{13}$$

The scale factor is set to unity at the end of inflation.

We set the initial conditions for our simulations by evolving all modes forward from the end of inflation, via Eqs. (10) to (13), until shortly before the perturbative description begins to break down. (Strictly, the Schrödinger-Poisson equations only become valid a few e-folds after the end of inflation. A full treatment would follow the analysis of Ref. [24] through this initial phase, but the change in the power spectrum is small and effectively absorbed into the definition of the spectrum and the parameter A.) The $\tilde{A}_k$ values are set via the power spectrum at the end of inflation. On superhorizon scales, they are time independent Fourier components of a random Gaussian field with amplitude $\alpha k^{1/2}/R_k(a=1)$. The constant α is set such that, at $a=1$, $\mathrm{rms}(\delta) \simeq 0.01$ on comoving distance scales of $0.02/H_{\mathrm{end}}$. This large value is chosen for computational convenience in this initial analysis.

Our ansatz sets the initial subhorizon fluctuation amplitude to zero. In Eq. (10), when $x \gtrsim 1$, δ_k oscillates, but does not grow. When $x \lesssim 1$, $\delta_k \propto a$. As inflation ends, $x \simeq 1$ for k_{horizon}, and subhorizon modes are initially oscillatory. Consequently, it can be shown that, unless the initial subhorizon power spectrum grows as fast as k^8, it will be subdominant when nonlinearity sets in. The initial spectrum and its evolution are shown in Fig. 1.

Modes which are mildly superhorizon at the end of inflation become nonlinear first, so our simulation volume must be initially superhorizon. Note, this also occurs in large cosmological N-body simulations; the Schrödinger-Poisson system is a similar Newtonian limit. Likewise, the breakdown of the perturbative description resembles the onset of nonlinearity during structure formation—the gravitational potential remains small, but density perturbations become large. Here, we take $L_{\mathrm{box}} = 10/H_{\mathrm{end}}$, in comoving units. For this choice, the full box is subhorizon when $a \gtrsim 10^2$, but all structure formation occurs on much smaller scales.

Results.—We focus on the density contrast $\delta = (\rho - \bar{\rho})/\bar{\rho}$, where $\bar{\rho}$ is the average density. Previous perturbative treatments of the inflaton field break down when $\delta \lesssim 1$, but here, we numerically evolve the inflaton condensate well into the nonlinear regime, reaching a maximum density contrast of 600 on a 512^3 grid. This corresponds to a scale factor of 200, which itself corresponds to $\gtrsim$3000 oscillations of the homogeneous background solution, and since $t \propto a^{3/2}$, most of these oscillations happen near the end of our simulation. We vary the time step to match the relevant dynamical timescale, but a full relativistic simulation would need to resolve all the oscillations.

Figure 2 shows both the maximum value and root mean square of δ in the simulation volume as a function of the scale factor for a given realization of our model. We compare the full nonlinear result to the purely perturbative evolution of Eq. (10) using the same initial configuration. The root mean square of the density contrast in the simulations clearly departs from the linear result. Moreover, the peak densities correspond to structures with density contrasts of $\mathcal{O}(10^2)$ that have broken away from the "Hubble flow." Note that these results also serve to verify the solver, which we also tested by extracting and plotting individual modes with $\delta \ll 1$ and matching them to the perturbation evolution.

Figure 3 shows a specific configuration shortly before the phase-gradient condition is violated. Qualitatively, the peaks are typically aspherical and occur at the intersections of a growing weblike network of overdensities, strongly reminiscent of those seen in standard cosmological structure simulations. The phase-gradient condition tends to be violated first at large overdensities; it is here that velocities

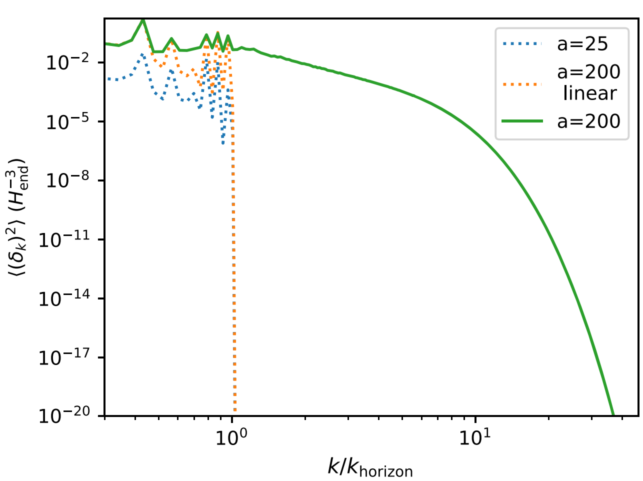

FIG. 1. Power spectrum of the density contrast at the beginning and end of the simulation. Scales are given relative to the horizon scale at the end of inflation.

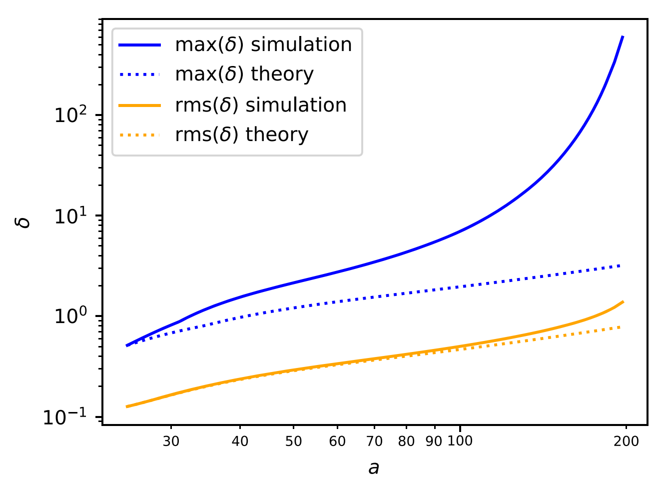

FIG. 2. Root mean square and maximum density contrast, measured at comoving distance scale $\simeq 0.02/H_{\mathrm{end}}$; the scale factor a is unity at the end of inflation. The results diverge as δ increases and perturbation theory breaks down.

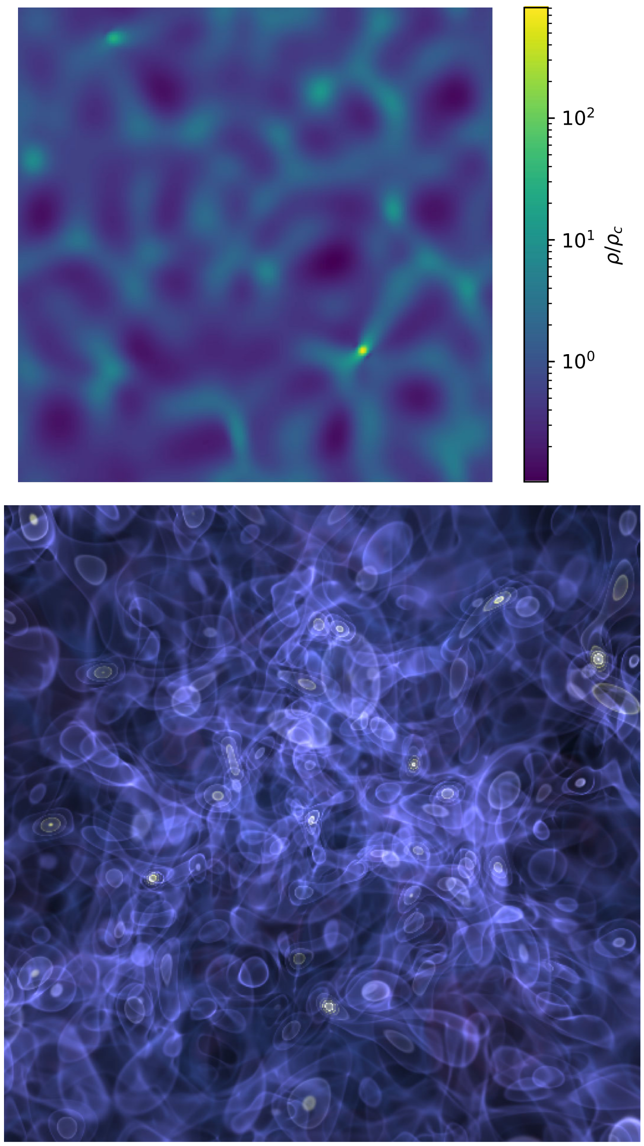

FIG. 3. Simulation results for an initial perturbation of amplitude $\sim 10^{-2}$; comoving simulation box size of 10 times postinflationary Hubble radius; when the Universe has expanded by a factor of $a = 200$ since the end of inflation. Top: The density ρ along a slice including the point of highest density. Bottom: Volume rendering of a subset of the box; blue regions $\delta \sim 1$; yellow and white regions $\delta \sim 10$–100.

will be largest. These breakdowns are localized and do not immediately "propagate" to the wider grid; running the code past the point at which they become manifest shows the further development of the web.

Consequently, this analysis confirms expectations that a pure inflaton condensate will fragment gravitationally if no other processes (such as resonance or prompt reheating) disrupt it earlier. Moreover, it demonstrates that Schrödinger-Poisson solvers can be used to investigate this previously unexplored regime of nonlinear dynamics in the postinflationary Universe.

Discussion.—This is the first exploration of nonlinear gravitational dynamics in the primordial dark age following inflation in scenarios without resonance [59]. We show that this phase is well described by the Schrödinger-Poisson equation, solving it numerically to demonstrate the nonlinear evolution and fragmentation of the inflaton field.

To calibrate the physical scales, if inflation ends at an energy density of $(10^{16}\ \mathrm{GeV})^4$, a single postinflationary horizon volume contains a mass of a few grams [60]. A long nonlinear phase could produce collapsed objects with substantially larger masses, but at scales that are still likely to be far too small for the resulting overdensities to leave a direct imprint on the Universe after thermalization. However, there are several ways in which this phase can have observable consequences. In particular, for any inflationary model, the "matching" between present-day and primordial scales depends on the reheating history, and this has a small but potentially detectable impact on the observable perturbation spectrum [44–46,61]. Moreover, in curvaton scenarios, the duration of the postinflationary "matter dominated" phase is a key parameter [62,63].

If reheating occurs via simple couplings between the inflaton and other species [21], particle production scales with the square of the local density and is enhanced by large inhomogeneities. In addition to thermalization, many possible dark matter populations can be (over)produced during the primordial dark age. In some cases, heavy relics overclose the Universe if the thermalization temperature is high (e.g., [64]); in others, dark matter production directly involves the postinflationary dynamics [48–56] and will be significantly affected by the fragmentation of the condensate.

Collapsing overdensities generically source gravitational waves [65–67] and nonlinear phases in the early Universe can generate stochastic gravitational wave backgrounds [59,68,69]. Typical accelerations and the resulting amplitudes produced via gravitational collapse are naively smaller than those from explosive resonance, but more speculatively, this new phase of nonlinear dynamics provides another channel for the production of a primordial gravitational wave background.

We performed simulations for a range of choices for the initial spectrum, and the outcomes did not depend strongly on the ansatz used. Higher resolution simulations will be needed to explore the detailed dynamics of the collapsed structures that form after the inflaton condensate fragments, which may include solitons and dynamical oscillonlike structures [11–15]. More sophisticated numerical strategies will allow the nonlinear phase to be investigated in detail.

Many lines of enquiry present themselves. Results for specific inflationary scenarios can be considered, with the initial conditions for the numerical solver propagated forward from the inflationary phase via perturbation theory [23,24], along with scenarios where the Compton wavelength is not similar to the comoving horizon size as inflation ends.

Solving the full Einstein equations for the overall evolution is prohibitively expensive, and there is no reason to expect models with simple initial perturbation spectra to produce primordial black holes. However, more complex scenarios suspected of forming singularities could be examined by using the output of a Schrödinger-Poisson simulation to initialize a relativistic solver [60,70–74].

The inflationary phase is a conjecture regarding physics outside the Standard Model, so the couplings between the inflationary sector and the matter content of the present Universe are entirely unknown. Consequently, while resonance has been extensively studied, there are no *a priori* grounds for determining whether it drives rapid thermalization, circumventing the nonlinear phase described here, but this is an important topic for future work. Likewise, in models where quanta of the inflation field are resonantly produced [15], the postresonance Universe may still be describable using Schrödinger-Poisson dynamics.

Finally, we are considering a class of inflationary models for which "structure formation" takes place in the early Universe as well as via the canonical postrecombination growth of galactic halos and many of the analytical and numerical tools used to describe the latter are likely to offer insight into the former.

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