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Holography in de Sitter Space via Chern-Simons Gauge Theory

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(Received 20 October 2021; revised 6 January 2022; accepted 21 April 2022; published 18 July 2022)

In this Letter, we propose a holographic duality for classical gravity on a three-dimensional de Sitter space. We first show that a pair of SU(2) Chern-Simons gauge theories reproduces the classical partition function of Einstein gravity on a Euclidean de Sitter space, namely $\mathbb{S}^3$, when we take the limit where the level k approaches -2. This implies that the conformal field theory (CFT) dual of gravity on a de Sitter space at the leading semiclassical order is given by an SU(2) Wess-Zumino-Witten model in the large central charge limit $k \to -2$. We give another evidence for this in the light of known holography for coset CFTs. We also present a higher spin gravity extension of our duality.

DOI: 10.1103/PhysRevLett.129.041601

Introduction.—Holography has been one of the most promising ideas that provide nonperturbative formulations of quantum gravity [1]. This approach has been extremely successful for holography in anti–de Sitter (AdS) space, namely the AdS/CFT correspondence [2]. On the other hand, to understand how the present Universe has been created, we need a complete formulation of quantum gravity in de Sitter (dS) space, instead of AdS. Nevertheless, we are still lacking understanding of holography in dS space, so-called dS/CFT correspondence [3–5] (see also Refs. [6–8]), though there has been a concrete proposal in four-dimensional higher spin gravity [9] and interesting recent progress in the light of the dS/dS correspondence [10–13], holographic entanglement entropy [14–17], and holography in dS static patch [18,19]. Especially, we are missing the dual conformal field theory (CFT) which lives on the past-future boundary of de Sitter space in Einstein gravity. This Letter is aimed at presenting a solution to this fundamental problem for three-dimensional dS.

The three-dimensional de Sitter space is special in that it is described by a Chern-Simons gauge theory [20] and that it is expected to be dual to a two-dimensional CFT assuming the standard idea of dS/CFT. The Chern-Simons description of gravity on $\mathbb{S}^3$, which is an Euclidean counterpart of de Sitter space, is described by a pair of SU(2) Chern-Simons gauge theories [20]. Moreover, it is well known that an SU(2) Chern-Simons theory is equivalent to conformal blocks of the SU(2) Wess-Zumino-Witten (WZW) model [21], which has often been regarded as an example of holography. By combining these observations, it is natural to suspect that the gravity on $\mathbb{S}^3$ and its Lorentzian continuation, i.e., de Sitter space, is dual to the SU(2) WZW model or its related cousins.

After a little consideration, however, we are immediately led to a puzzle as follows. Since the classical limit of the Einstein gravity on $\mathbb{S}^3$ or de Sitter space is given by the large level limit $k \to \infty$ (see Refs. [22–28] for various studies of this limit), the central charge c of the dual SU(2) WZW model at level k approaches to the finite value $c = 3k/(k+2) \to 3$ in this limit. On the other hand, the standard idea of dS/CFT [3,5] tells us that the classical gravity is dual to the large central charge limit of a CFT. In what follows, as the main result in this Letter, we will show that, in the large central charge limit $k \to -2$ of the SU(2) WZW model, the dual Chern-Simons gravity is able to reproduce the results of classical gravity on $\mathbb{S}^3$. By combining this observation with a de Sitter generalization of the conjectured higher spin AdS/CFT duality [29], we will resolve the above puzzle and obtain a concrete dS/CFT in the three-dimensional case.

Chern-Simons gravity on $\mathbb{S}^3$.—The Einstein gravity on $\mathbb{S}^3$ is equivalent to two copies of classical SU(2) Chern-Simons gauge theories, whose action is given by

$$I_{\rm CSG} = I_{\rm CS}[A] - I_{\rm CS}[\bar{A}],$$
$$I_{\rm CS}[A] = -\frac{k}{4\pi}\int_{\mathcal{M}} {\rm Tr}[A \wedge dA + \frac{2}{3} A \wedge A \wedge A], \quad (1)$$

where A and $\bar{A}$ are the one-form SU(2) gauge potentials. The level k is inversely proportional to the three-dimensional

Newton constant G_N. The partition function of a single SU(2) Chern-Simons theory with a Wilson loop in the spin-j representation (denoted by R_j), is given by $\mathcal{S}_j^0$ [21], where $\mathcal{S}$ is the $\mathcal{S}$ matrix of modular transformation of SU(2) WZW model,

$$\mathcal{S}_j^l = \sqrt{\frac{2}{k+2}} \sin\left[\frac{\pi}{k+2}(2j+1)(2l+1)\right]. \qquad (2)$$

Therefore, the total partition function of the Chern-Simons theory (1) for the three-dimensional gravity is evaluated as

$$Z_{\mathrm{CSG}}[\mathbb{S}^3, R_j] = |\mathcal{S}_0^j|^2, \qquad (3)$$

where we assumed that the Wilson loop is symmetric between the two SU(2) gauge groups.

Moreover, when two Wilson loops, each in the R_j and R_l representation, are linked, the partition function of the Chern-Simons gravity reads

$$Z_{\mathrm{CSG}}[\mathbb{S}^3, L(R_j, R_l)] = |\mathcal{S}_j^l|^2. \qquad (4)$$

On the other hand, when two Wilson loops are not linked with each other, we obtain

$$Z_{\mathrm{CSG}}[\mathbb{S}^3, R_j, R_l] = \left|\frac{\mathcal{S}_0^j \mathcal{S}_0^l}{\mathcal{S}_0^0}\right|^2. \qquad (5)$$

Note that the above partition functions are for the full quantum Chern-Simons theory, and thus we expect they include quantum gravity effects, which will be suppressed in the large k limit.

Holographic limit for dS/CFT.—Motivated by the standard version of dS/CFT correspondence in [5], where Einstein gravity limit of three-dimensional de Sitter space is given by the large central charge limit $c \to i\infty$, we argue the following relation between the SU(2) WZW model and the gravity on $\mathbb{S}^3$:

$$c = \frac{3k}{k+2} = ic^{(g)},$$
$$h_j = \frac{j(j+1)}{k+2} = ih_j^{(g)}, \qquad (6)$$

where c and h_j are, respectively, the central charge and the chiral conformal dimension of a primary field in the SU(2) WZW model at level k, respectively, while the quantities $c^{(g)}$ and $h_j^{(g)}$ are their gravity counterparts and are real valued. In the gravity, the radius of $\mathbb{S}^3$, written as L, is related to the central charge via the de Sitter counterpart of the well-known relation [5,30]

$$c^{(g)} = \frac{3L}{2G_N}. \qquad (7)$$

The energy E_j in this gravity dual to the Wilson loop R_j is simply related to the conformal dimension via

$$E_j = \frac{2h_j^{(g)}}{L}. \qquad (8)$$

In the semiclassical gravity regime $L/G_N \gg 1$, we consider $k \to -2$ limit, which is more precisely described by

$$k = -2 + \frac{6i}{c^{(g)}} + O\left(\frac{1}{c^{(g)2}}\right). \qquad (9)$$

In addition it is useful to note

$$1 - 8G_N E_j = 1 - \frac{24h_j^{(g)}}{c^{(g)}} \simeq (2j+1)^2. \qquad (10)$$

Therefore, the Chern-Simons partition function on $\mathbb{S}^3$ with a single Wilson loop (3) is evaluated as follows (in the semiclassical limit $c^{(g)} \gg 1$):

$$Z_{\mathrm{CSG}}[\mathbb{S}^3, R_j] \simeq \frac{c^{(g)}}{12} \exp\left[\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_j}\right]. \qquad (11)$$

Similarly, the partition function (4) on $\mathbb{S}^3$ with two linked Wilson loops inserted is estimated by

$$Z_{\mathrm{CSG}}[\mathbb{S}^3, L(R_j, R_l)] \simeq \frac{c^{(g)}}{12} \exp\left[\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_j}\sqrt{1-8G_N E_l}\right]. \qquad (12)$$

For unlinked two Wilson lines, we obtain from (5),

$$Z_{\mathrm{CSG}}[\mathbb{S}^3, R_j, R_l] \simeq \frac{c^{(g)}}{12} \exp\left[\frac{\pi c^{(g)}}{3}\left(\sqrt{1-8G_N E_j} + \sqrt{1-8G_N E_l} - 1\right)\right]. \qquad (13)$$

Notice that in the above we have assumed the limit $k \to -2$, which looks quite different from the semiclassical limit of the Chern-Simons gauge theory. To see that our new limit gives a correct answer, we will compare the above results with those expected from the direct Einstein gravity calculations in the following.

Gravity on de Sitter space.—The Euclidean de Sitter black hole solution is given by

$$ds^2 = L^2\left[(1-8G_N E_j - r^2)d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right], \qquad (14)$$

where E_j is the energy of an excitation [31]. The black hole horizon is at $r = \sqrt{1-8G_N E_j}$ and the requirement of smoothness at the horizon determines the periodicity

$$\tau \sim \tau + \frac{2\pi}{\sqrt{1-8G_N E_j}}. \tag{15}$$

On the other hand, the angular coordinate ϕ obeys the periodicity $\phi \sim \phi + 2\pi$ and there is a conical singularity at $r = 0$. The black hole entropy reads

$$S_{\rm BH} = \frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_j}. \tag{16}$$

It is useful to introduce the coordinate θ by

$$r = \sqrt{1-8G_N E_j}\sin\theta \qquad \left(0 \le \theta \le \frac{\pi}{2}\right), \tag{17}$$

which leads to the metric

$$ds^2 = L^2[d\theta^2 + (1-8G_N E_j)(\cos^2\theta d\tau^2 + \sin^2\theta d\phi^2)]. \tag{18}$$

Then we evaluate the gravity action

$$I_G = -\frac{1}{16\pi G_N}\int \sqrt{g}(R-2\Lambda), \tag{19}$$

where $\Lambda = 1/L^2$. This leads to

$$I_G = -\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_j}, \tag{20}$$

whose semiclassical gravity partition function $Z_G = \exp[-I_G]$ agrees with the Chern-Simons result (11).

Let us introduce the Cartesian coordinates

$$\begin{aligned} X_1 &= \cos\theta\cos(\sqrt{1-8G_N E_j}\tau), \\ X_2 &= \cos\theta\sin(\sqrt{1-8G_N E_j}\tau), \\ X_3 &= \sin\theta\cos(\sqrt{1-8G_N E_j}\phi), \\ X_4 &= \sin\theta\sin(\sqrt{1-8G_N E_j}\phi). \end{aligned} \tag{21}$$

Then the sphere $\sum_{i=1}^{4}(X_i)^2 = L^2$ is described by the metric (18). The insertion of the single Wilson line R_j corresponds to a deficit angle $\delta_j = 2\pi - 2\pi\sqrt{1-8G_N E_j}$ at $\theta = 0$, depicted as the red circle in Fig. 1.

We can realize the second Wilson loop at $\theta = \pi/2$ linking with the first one by identifying the coordinate τ as

$$\tau \sim \tau + \frac{2\pi\sqrt{1-8G_N E_l}}{\sqrt{1-8G_N E_j}}. \tag{22}$$

This is depicted as the green circle in Fig. 1, where the deficit angle $\delta_l = 2\pi - 2\pi\sqrt{1-8G_N E_l}$ is present. Finally, the gravity action for this geometry is estimated as

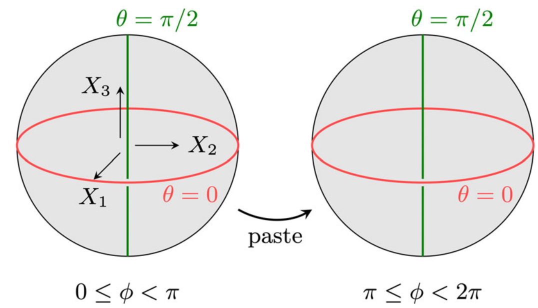

FIG. 1. The north (left) and south (right) hemisphere with two linked Wilson lines (red and green).

$$I_G = -\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_j}\sqrt{1-8G_N E_l}, \tag{23}$$

which again reproduces the leading part of the Chern-Simons result (12) in the semiclassical limit.

Higher spin gravity.—The Chern-Simons theory enables us to construct a broader class of three-dimensional gravity theories, namely, higher spin gravity. A pair of SU(N) Chern-Simons theories at level k describes a three-dimensional gravity with spin-s fields for $s = 2, 3, \ldots, N$.

The central charge of the SU(N) WZW model at level k reads

$$c = \frac{k(N^2-1)}{k+N}. \tag{24}$$

The chiral conformal dimension of a primary in the representation specified by a weight vector $\lambda = \sum_{i=1}^{N-1}\lambda_i\omega_i$ is given by

$$h_\lambda = \frac{(\lambda, \lambda+2\rho)}{2(k+N)}, \tag{25}$$

where $\rho = \sum_{i=1}^{N-1}\omega_i$. The weight lattice is generated by the basis $\{\omega_1, \ldots, \omega_{N-1}\}$ and its inner product is denoted by $(*, *)$. Here we follow the convention in [32]. The modular $\mathcal{S}$ matrix reads

$$\mathcal{S}_\lambda^\mu = K\sum_{w\in W}\epsilon(w)\exp\left[-\frac{2\pi i}{k+N}(w(\lambda+\rho), \mu+\rho)\right], \tag{26}$$

where W is the Weyl group and K is a constant fixed by the unitary constraint $\mathcal{S}\mathcal{S}^\dagger = 1$.

Now we analytically continue the level as we did in the SU(2) case, $c = ic^{(g)}$ and $h_\lambda = ih_\lambda^{(g)}$, which leads to

$$k = -N + N(N^2-1)\frac{i}{c^{(g)}} + O\left(\frac{1}{c^{(g)2}}\right). \tag{27}$$

Let us evaluate $\mathcal{S}_0^0$, which gives the vacuum partition function $Z_{\rm CSG}[\mathbb{S}^3]$. By using the known relation

$(\rho,\rho) = N(N^2-1)/12$, the partition function of the SU(N) Chern-Simons gravity with linked Wilson loops in the limit (27) looks like

$$Z_{\mathrm{CSG}}[\mathbb{S}^3, L(R_\lambda, R_\mu)] = |\mathcal{S}^\mu_\lambda|^2 \sim \exp\left[\frac{\pi c^{(g)}}{3}\frac{(\lambda+\rho,\mu+\rho)}{(\rho,\rho)}\right]. \tag{28}$$

It is straightforward to confirm that this reproduces the previous result (12) if we set $N=2$. Moreover, it is useful to note that this group theoretical argument explains the partition function with unlinked Wilson loops R_j and R_l, given by (5). Indeed, by setting $\lambda = \lambda_j + \lambda_l$ and $\mu = 0$, we can rewrite $(\lambda+\rho,\mu+\rho) = (\lambda_j+\rho,\rho) + (\lambda_l+\rho,\rho) - (\rho,\rho)$.

As in the $N=2$ case, we will show below that the partition function (28) of the SU(N) Chern-Simons gravity computed from the $k \to -N$ limit of the SU(N) WZW model equals that of the corresponding higher spin gravity in the classical limit, i.e., the large level limit. The configuration of the SU(N) gauge fields describing a conical geometry can be constructed in a similar manner to the AdS case presented in [33]. We find it convenient to use the $\bar{A}=0$ gauge, where the solution of A is given by

$$A = (hb^2\bar{h})^{-1} d(hb^2\bar{h}), \tag{29}$$

with parameters

$$\begin{aligned} b &= \prod_{i=1}^{N} \exp[\rho_i e_{i,i}] \qquad \left(\rho_i \equiv \frac{N+1}{2} - i\right), \\ h &= \prod_{i=1}^{\lfloor \frac{N}{2} \rfloor} \exp\left[-(e_{2i-1,2i} - e_{2i-1,2i})(n_i\phi + \tilde{n}_i\tau)\right], \\ \bar{h} &= \prod_{i=1}^{\lfloor \frac{N}{2} \rfloor} \exp\left[(e_{2i-1,2i} - e_{2i-1,2i})(n_i\phi - \tilde{n}_i\tau)\right]. \end{aligned} \tag{30}$$

Here $e_{i,j}$ are $N\times N$ matrices with elements $(e_{i,j})^l_k = \delta_{ik}\delta^l_j$.

The on-shell action (1) for the gauge configuration can be evaluated as

$$I_{\mathrm{CSG}} = -\frac{\pi}{G_N}\frac{\sum_{i=1}^{\lfloor \frac{N}{2} \rfloor} n_i\tilde{n}_i}{(\rho,\rho)}, \tag{31}$$

where we use the relation between the Chern-Simons level and the Newton constant in the higher spin gravity,

$$k = \frac{L}{8G_N(\rho,\rho)}. \tag{32}$$

Let us rewrite the eigenvalues as $n_1 \geq n_2 \ldots, \tilde{n}_1 \geq \tilde{n}_2 \ldots$ and set $n_i = -n_{N+1-i}$, $\tilde{n}_i = -\tilde{n}_{N+1-i}$ for $i > \lfloor N/2 \rfloor$ [34]. If we require $n_i \neq n_j$ and $\tilde{n}_i \neq \tilde{n}_j$ for $i \neq j$, which generically corresponds to the diagonalizability of the matrix, then we could set $n_i = \lambda_i + \rho_i$, $\tilde{n}_i = \mu_i + \rho_i$. In this representation, with the identification (7), we can rewrite (31) as

$$I_{\mathrm{CSG}} = -\frac{\pi c^{(g)}}{3}\frac{(\lambda+\rho,\mu+\rho)}{(\rho,\rho)}. \tag{33}$$

Hence the on-shell partition function $Z_{\mathrm{CSG}} = e^{-I_{\mathrm{CSG}}}$ for the higher spin gravity agrees with the expression (28) obtained from the modular $\mathcal{S}$ matrix as we promised.

Entanglement and black hole entropy.—Let us turn to the calculation of entanglement entropy in the gravity on $\mathbb{S}^3$. We choose a subsystem A to be a disk on the surface $\mathbb{S}^2$, which separates $\mathbb{S}^3$ into two hemispheres. We write the boundary circle of A as Γ_A. In the replica calculation of entanglement entropy, we introduce a cut along Γ_A on $\mathbb{S}^3$ and take its n-fold cover to obtain $\mathrm{Tr}[(\rho_A)^n]$. The replica calculation in Chern-Simons theory was performed in [35] to read off the topological entanglement entropy [36,37] in terms of modular matrices. In the presence of a Wilson loop R_μ, which is linked with Γ_A, we obtain (refer to [38] for an AdS counterpart)

$$S_A = \log|\mathcal{S}^\mu_0|^2 = \frac{\pi c^{(g)}}{3}\frac{(\rho,\mu+\rho)}{(\rho,\rho)}. \tag{34}$$

For the Einstein gravity ($N=2$) with a Wilson loop R_j, it takes the following form:

$$S_A = \log|\mathcal{S}^j_0|^2 = \frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_j}. \tag{35}$$

This indeed coincides with the de Sitter black hole entropy (16). It is straightforward to extend the above result to the topological pseudo entropy [39,40].

Discussions: dS/CFT *interpretation.*—We have shown that the limit $k\to -2$ for two copies of the SU(2) Chern-Simons gauge theories, where the central charge of its dual SU(2) WZW model gets infinitely large $c \to i\infty$, reproduces the Einstein gravity on $\mathbb{S}^3$. More generally, the large central charge limit $k \to -N$ of the SU(N) WZW model corresponds to the classical limit of the SU(N) higher spin gravity on $\mathbb{S}^3$. We argue that this is a manifestation of the (Euclidean version of) dS/CFT correspondence.

One may worry that this might contradict the standard fact that the classical limit of higher spin gravity on $\mathbb{S}^3$ is given by not finite k, but the large k limit of two copies of SU(N) Chern-Simons theory. To reconcile this tension, let us consider the following coset CFT, called the W_N-minimal model:

$$\frac{\mathrm{SU}(N)_k \times \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}}, \tag{36}$$

which has the central charge

$$c = (N-1)\left(1 - \frac{N(N+1)}{(N+k)(N+k+1)}\right). \tag{37}$$

In [29], this model is argued to be dual to the higher spin gravity on AdS_3 (Vasiliev theory [41]) with two complex scalar fields if we take the 't Hooft limit

$$N \to \infty, \qquad k \to \infty, \qquad \hat{\lambda} = \frac{N}{N+k} = \text{fixed}. \tag{38}$$

This higher spin gravity has the symmetry $\text{hs}[\hat{\lambda}]$, which enhances to $W_\infty[\hat{\lambda}]$ at the asymptotic boundary [42,43]. In our limit $k \to -N$, the contribution to the total central charge of the coset is dominated by the $SU(N)_k$ part and thus the leading contribution comes from the (nonchiral) $SU(N)$ WZW model, which is essentially the same model we have studied in the above. Interestingly, the triality [44] relates three different values of the previous parameters $(k, N, \hat{\lambda})$ via the following two duality relations:

$$\text{(i)}\ (k', N', \hat{\lambda}') = \left(-2N - k - 1, N, -\frac{N}{N+k+1}\right),$$

$$\text{(ii)}\ (k', N', \hat{\lambda}') = \left(\frac{1-N}{N+k}, \frac{N}{N+k}, N\right). \tag{39}$$

If we apply the duality (ii) to the $k \to -2$ limit (9) at $N = 2$ (see also Refs. [45,46] for a similar continuation), we find

$$(k', N', \hat{\lambda}') \simeq \left(i\frac{c^{(g)}}{6}, -i\frac{c^{(g)}}{3}, 2\right). \tag{40}$$

Thus, this theory has $W_\infty[2]$ symmetry, i.e., Virasoro symmetry, which is indeed expected for the Einstein gravity. We can generalize this to the $k \to -N$ limit of the $SU(N)$ theory, for which the duality (ii) predicts $W_\infty[N] \simeq W_N$ symmetry with the level infinitely large as expected for the classical $SU(N)$ higher spin gravity. In this way, our dS/CFT example is consistent with an extension of earlier results, at least in the leading order.

Finally, we would like to discuss the spectrum of the $SU(N)_k$ WZW model. Consider the simplest case with $N = 2$. For integer k, the unitary representation is given with $j = 0, 1/2, 1, \ldots, k/2$. However, now the level k is complex as in (9), and moreover, the WZW model is not unitary due to the imaginary central charge. As discussed above, the SU(2) WZW model can be regarded as a part of an analytic continuation of the Virasoro minimal model. Thus, it is natural to use the representation labeled by $j = 0, 1/2, 1, \ldots$ without the upper bound of j. The dual geometry is constructed by the gauge configuration (29) with (30). Requiring the trivial holonomy conditions as in [33], the allowed parameter becomes $n_1 - 1/2 = 0, 1/2, 1, \ldots$, which is consistent with the CFT spectrum. We can see that situation is similar for generic N, as will be explained in the upcoming paper [47].

This type of analytic continuation has been carefully analyzed in [44]. The coset CFT (36) has the symmetry of W_N algebra, but it is useful to consider a larger algebra $W_\infty[\hat{\lambda}]$ with a complex central charge. The degenerate representations of $W_\infty[\hat{\lambda}]$ algebra have been also analyzed in, e.g., [44,48]. The algebra can be truncated to W_N algebra at $\hat{\lambda} = N$, which is the one used in this Letter. When k is also positive integer, then a further truncation is possible, leading to the W_N-minimal model.

The W_N-minimal model has long believed to be equivalent to the coset model (36) with positive integer N, k, which was proven rather recently in [49]. Similarly, it was recently shown in [50] that the coset model (36) with positive integer N but generic k is equivalent to Toda CFT with generic central charge. The Toda description is useful to compute correlation functions [51,52]. Let us think about the simplest example with $N = 2$ again, then the coset model (36) reduces to the Liouville CFT. The Liouville CFT is described by a bosonic field ϕ with background charge $Q = b + 1/b$. The central charge is

$$c = 1 + 6Q^2 = 1 - \frac{6}{(k+2)(k+3)}, \tag{41}$$

which also leads to the relation to the level k of the coset model (36). In particular, the limit $k \to -2$ is realized by $b \to 0$ (or $b \to \infty$). The vertex operators are of the form $V_\alpha = \exp(2\alpha\phi)$, where $\alpha = Q/2 + ip$ or $\alpha = \alpha_{r,s}$ with $\alpha_{r,s} = [b(1-r) + b^{-1}(1-s)]/2 \quad (r, s = 1, 2, \ldots)$. The modular $\mathcal{S}$ matrix between these two types is [53]

$$\mathcal{S}^p_{(r,s)} \propto \sinh(2\pi rpb) \sinh(2\pi sp/b). \tag{42}$$

Setting $p = (iQ/2)(2j+1)$, $s = 2l + 1$, and $ic^{(g)} \simeq 6/b^2$ with $b \to 0$, we find

$$\left|\mathcal{S}^{\frac{iQ}{2}(2j+1)}_{(r,s)}\right|^2 \sim e^{\frac{\pi}{3}c^{(g)}\sqrt{1-8G_N E_l}\sqrt{1-8G_N E_j}}, \tag{43}$$

which reproduces the gravity result (12). This calculation is equivalent to our previous one, where (2) is applied to noninteger k and thus justifies our analytical continuation. Similar analysis is possible for generic N [47].

It will be interesting to examine correlation functions, quantum gravity corrections, and a Lorentzian continuation explicitly, which we plan to come back soon [47].

We are grateful to Yasunori Nomura for useful discussions. This work was supported by JSPS Grant-in-Aid for Scientific Research (A) No. 21H04469, Grant-in-Aid for Transformative Research Areas (A) No. 21H05182, No. 21H05187, and No. 21H05190. T. T. is supported by the Simons Foundation through the "It from Qubit" Collaboration, Inamori Research Institute for Science and

World Premier International Research Center Initiative (WPI Initiative) from the Japan Ministry of Education, Culture, Sports, Science and Technology (MEXT). The work of Y. H. was supported in part by the JSPS Grant-in-Aid for Scientific Research (B) No. 19H01896. The work of T. N. was supported in part by the JSPS Grant-in-Aid for Scientific Research (C) No. 19K03863.

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