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Static Black Binaries in de Sitter Space

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We construct the first four-dimensional multiple black hole solution of general relativity with a positive cosmological constant. The solution consists of two static black holes whose gravitational attraction is balanced by the cosmic expansion. These static binaries provide the first four-dimensional example of nonuniqueness in general relativity without matter.

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Introduction.—Black holes are famously featureless. This idea is embodied by the no-hair theorems, which state in essence that stationary black holes are uniquely characterized by their mass, angular momentum, and charge [1–11].

It should be noted that there are many situations where black hole uniqueness, as we have expressed it, is known to be violated. A well-known example involves multihorizon configurations of charged, extremal black holes [12,13]. Other examples include higher dimensions [14], anti-de Sitter asymptotics [15–17], or exotic matter like classical Yang-Mills fields, complex scalars, and Proca fields [18–20].

Additionally, there are some mathematical gaps in fully establishing black hole uniqueness, even in the more limited case of four-dimensional pure gravity in flat space. Indeed, asymptotically flat multi-Kerr black holes, where their gravitational attraction might be balanced by spin-spin interactions, have not been ruled out (see, e.g., [21–33] for attempted constructions that yield singular configurations). Though for static solutions, a classic theorem due to [34–36] precludes the existence of regular asymptotically flat multiple black holes.

Despite these (and potentially more) counterexamples, there is currently no experimental or observational evidence that black hole nonuniqueness can be realized in astrophysical or cosmological contexts. Indeed, the no-hair theorems are fully consistent with observational results from the LIGO consortium [37].

However, the no-hair theorems assume that spacetime is asymptotically flat, a feature that is violated in our Universe at the longest scales by the presence of a cosmological constant [38–42]. The resulting cosmic expansion might balance out the gravitational attraction of two or more black holes, allowing multiple black holes to exist in static equilibrium. Such a configuration would share the same mass and angular momentum as some single-horizon black hole and therefore serve as a more realistic counterexample to black hole uniqueness.

The aim of this Letter is to demonstrate that such a multihorizon configuration indeed occurs within general relativity. We will focus on the simplest case with two equal-mass black holes that do not rotate or contain charge, but our results and methods can be straightforwardly generalized. We will first show that these black binaries can be anticipated using intuition from Newton-Hooke theory, and then construct these solutions by solving the Einstein equation numerically. Finally, we study the properties of these binaries in detail.

Our results, along with physical intuition, suggest that the static de Sitter binaries are dynamically unstable. Nevertheless, there remains a possibility that they can be stabilized with the introduction of charge or angular momentum. We will comment on this and other matters in the conclusions.

Before we continue, we mention some closely related work. Dynamical (i.e., out of equilibrium) multiple black holes in Einstein-Maxwell theory with a positive cosmological constant were found in [43]. The "rod structure" corresponding to our static binaries were anticipated and examined in detail in [44]. In [45], a novel mechanism for balancing multiple black holes was proposed. These constructions provide Ricci-flat, closed-form solutions for static binaries supported by expanding bubbles of nothing. Mechanically, these solutions behave similarly to the static binaries we find.

Finally, we mention the mathematical papers [46–48], which might seem to rule out the existence of static black binaries in de Sitter. We will show that the assumptions made in [46,47] do not apply, and that (for technical reasons) this conclusion from [48] is not correct.

Newton-Hooke.—Let us first set out to see if the aforementioned multiple black hole configurations are allowed within Newtonian gravity. We adopt geometrized units in which $c = G = k_B = \hbar = 1$.

Consider a configuration of N black holes with masses m_a, with $a = 1, \ldots, N$. For the Newtonian approximation to be valid, we assume that the distances between the black holes are much larger than their masses. We now include the effects of the cosmological constant $\Lambda \equiv 3/\ell^2 > 0$, where ℓ is the de Sitter length scale. Accordingly, we assume that the entire configuration of black holes lies within a distance much smaller than ℓ and consider the Newton-Hooke equations of motion [49,50]

$$m_a \frac{d^2 \boldsymbol{x}_a}{dt^2} - m_a \frac{\boldsymbol{x}_a}{\ell^2} = -\sum_{b \neq a}^{b=N} \frac{m_a m_b (\boldsymbol{x}_a - \boldsymbol{x}_b)}{|\boldsymbol{x}_a - \boldsymbol{x}_b|^3}, \qquad (1)$$

where $\boldsymbol{x}_a$ are the positions of the black holes.

Static solutions exist when

$$\frac{\boldsymbol{x}_a}{\ell^2} = \sum_{b \neq a}^{b=N} \frac{m_b (\boldsymbol{x}_a - \boldsymbol{x}_b)}{|\boldsymbol{x}_a - \boldsymbol{x}_b|^3}. \qquad (2)$$

Such solutions are known as “central configurations” and provide homothetic solutions of the Newtonian N-body problem that have applications to Newtonian cosmology. Equation (2) can also be obtained from the Dmitriev-Zeldovich equations [51] by using the scale factor $S(t) = e^{(t/\ell)}$, corresponding to a de Sitter background in “stead-state” coordinates [49,50,52,53].

Consider a central configuration with two equal mass black holes aligned along the z axis and separated by a distance d. That is, $N = 2$, $\boldsymbol{x}_1 = -\boldsymbol{x}_2 = (d/2)\boldsymbol{e}_z$, and $m_a = m_b = M$. Then Eq. (2) imposes

$$\frac{d^3}{\ell^3} = \frac{r_+}{\ell}, \qquad (3)$$

where $r_+ \equiv 2M$ is the Schwarzschild radius.

The requirements that the Newton-Hooke approximation should be valid and that the black holes are inside a single cosmological event horizon amount to

$$r_+ \ll d, \qquad d \ll \ell, \quad \text{and} \quad r_+ \ll \ell. \qquad (4)$$

If the distance between the black holes is given as in Eq. (3), then we see that the first two conditions above are satisfied if we assume the third, i.e., if the black holes are small enough. We therefore conclude that static de Sitter binaries with small black holes are consistent with Newton-Hooke theory.

For later use, we introduce the event horizon Hawking temperature $T_+ = (4\pi r_+)^{-1}$ and rewrite Eq. (3) as

$$\frac{d}{\ell} = \frac{1}{(4\pi \ell T_+)^{1/3}}. \qquad (5)$$

We will confirm that our numerical solutions to the Einstein equation satisfy this scaling in the appropriate limit.

Numerical construction.—We now construct static binaries in general relativity by numerically solving the Einstein equation with a positive cosmological constant:

$$R_{ab} = \frac{3}{\ell^2} g_{ab}, \qquad (6)$$

where R_{ab} is the Ricci tensor and g_{ab} is the metric tensor.

We use the DeTurck method, first introduced for general relativity in [54] and reviewed in [55,56]. This method provides a convenient way of addressing the issue of gauge invariance, which ultimately causes the Einstein equation [Eq. (6)] to yield a set of ill-posed, nonelliptic partial differential equations (PDEs).

The DeTurck method involves choosing any reference metric $\bar{g}$ with the same symmetries and causal structure as the solution we seek. In this case, our reference metric is static, contains two identical black holes and a cosmological horizon, and is axisymmetric. There is therefore a discrete $\mathbb{Z}_2$ symmetry, as well as two Killing vector fields $k = \partial/\partial t$ and $m = \partial/\partial\phi$. We further assume that the black holes and cosmological horizon are Killing horizons generated by k. Our choice of reference metric involves a combination of the Bach-Weyl solution with two identical black holes [57] (equivalent to the Israel-Khan solution [58,59] with two black holes) and the static patch of de Sitter space. Its design is detailed in the Supplemental Material [60].

We then write down the most general metric ansatz g that respects the desired symmetries and causal structure. In this case, the metric ansatz depends nontrivially on two coordinates (i.e., it is cohomogeneity-two, and will yield two-dimensional PDEs).

We then solve the Einstein-DeTurck equation

$$R_{ab} - \nabla_{(a}\xi_{b)} = \frac{3}{\ell^2} g_{ab}, \qquad (7)$$

where $\xi^a \equiv g^{bc}[\Gamma^a_{bc}(g) - \Gamma^a_{bc}(\bar{g})]$, and $\Gamma(\mathfrak{g})$ is the metric-preserving Christoffel connection associated to a metric $\mathfrak{g}$. Unlike the Einstein equation, the Einstein-DeTurck equation [Eq. (7)] yields a set of elliptic PDEs [54–56,61], which gives a well-posed boundary-value problem with appropriate physical boundary conditions.

The Einstein-DeTurck equation [Eq. (7)] is solved numerically. One complication is that the integration domain contains five boundaries: the $\mathbb{Z}_2$ reflection surface, the inner and outer portions of the symmetry axes, the black hole horizons, and the cosmological horizon. We handle this domain using patching techniques. This and other numerical methods we use are described in [56] and detailed in the Supplemental Material.

After solving Eq. (7), we must verify that the solution actually solves the Einstein equation, i.e., that $\xi = 0$, and is therefore not a Ricci soliton (for which $\xi \neq 0$). Under many circumstances [61,62], it can be proved that these unwanted Ricci solitons do not exist. Unfortunately, the present case is not one of these circumstances. Indeed, with a positive cosmological constant, Ricci solitons are known to exist (see, e.g., [63]). Nevertheless, ellipticity guarantees local uniqueness. That is, solutions with $\xi = 0$ cannot be arbitrarily close to solutions with $\xi \neq 0$, and thus the norm $\xi^a \xi_a$ can be monitored to identify whether our numerical discretization converges in the continuum to a Ricci soliton or to a true solution of the Einstein equation. In the Supplemental Material, we provide ample evidence that the numerical solutions we construct are *not* Ricci solitons.

Results.—Having numerical solutions corresponding to static black binaries in de Sitter, we can now describe their properties and compare the numerical results to Newton-Hooke theory when the latter is valid.

We expect to find agreement with Newton-Hooke theory when the black holes become sufficiently small, or alternatively, when the black hole temperature becomes sufficiently large $T_+\ell \gg 1$. In Fig. 1, we provide a log-log plot of the proper distance between the horizons of the two black holes along the symmetry axis $\mathcal{P}_\phi/\ell$, as a function of temperature $4\pi T_+\ell$. The solid black line is the scaling [Eq. (5)], and the blue dots are the numerical data. The agreement at large values of $T_+\ell$ shows the validity of the Newton-Hooke analysis and corroborates our numerical construction.

We have not managed to find solutions with large black holes (small $4\pi T_+\ell$). Because our solutions do not have regions of large curvature, there might be a "turning point" to a new branch of solutions. A similar phenomenon occurs for localized Kaluza-Klein black holes when the black holes are large relative to the Kaluza-Klein circle [54,55,64–83]. We leave the exploration of this region of parameter space for future work.

Let us now discuss black hole thermodynamics. For a central configuration containing N black holes inside the static patch of de Sitter, the covariant phase space formalism [84–95] shows that the following form of the first law of black hole mechanics holds:

$$\sum_{i=1}^{N} T_+^{(i)} dS_+^{(i)} = -T_c dS_c, \tag{8}$$

where T_c is the temperature of the cosmological horizon, and S_c is its entropy (i.e., horizon area). $T_+^{(i)}$ and $S_+^{(i)}$ are the same quantities, respectively, for the ith black hole. With $N = 2$ and equal-mass black holes, we find

$$2T_+ dS_+ = -T_c dS_c. \tag{9}$$

We have checked that our data satisfies this form of the first law to within 0.01%.

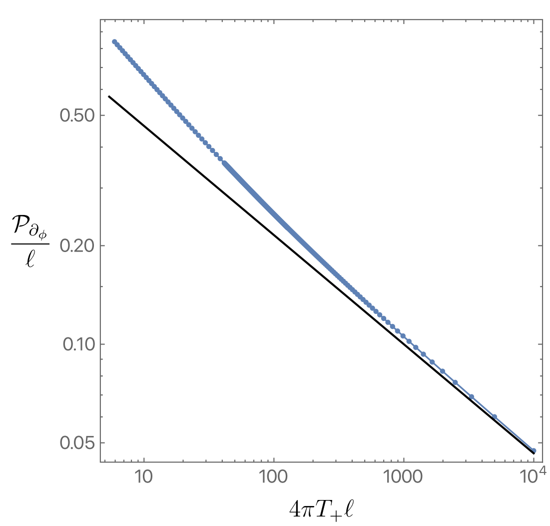

FIG. 1. Proper distance between the black hole horizons versus the black hole temperature. The solid black line shows the scaling [Eq. (5)] according to Newton-Hooke analysis and the blue dots show numerical data according to general relativity.

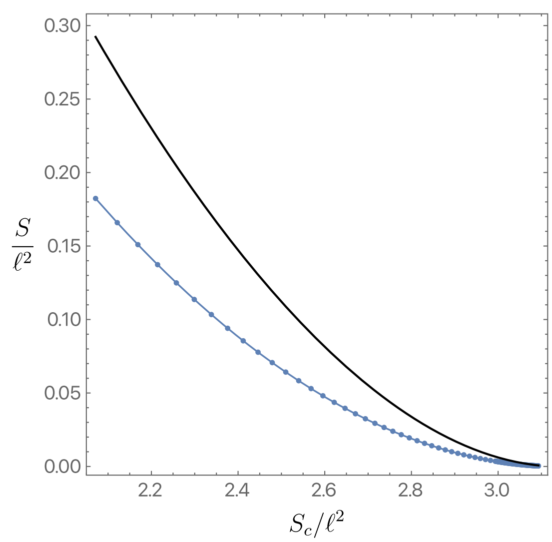

FIG. 2. Total black hole entropy versus the cosmological horizon entropy. The blue dots are numerical data for static binaries ($S = 2S_+$) and the solid black line is for the single Schwarzschild-de Sitter black hole.

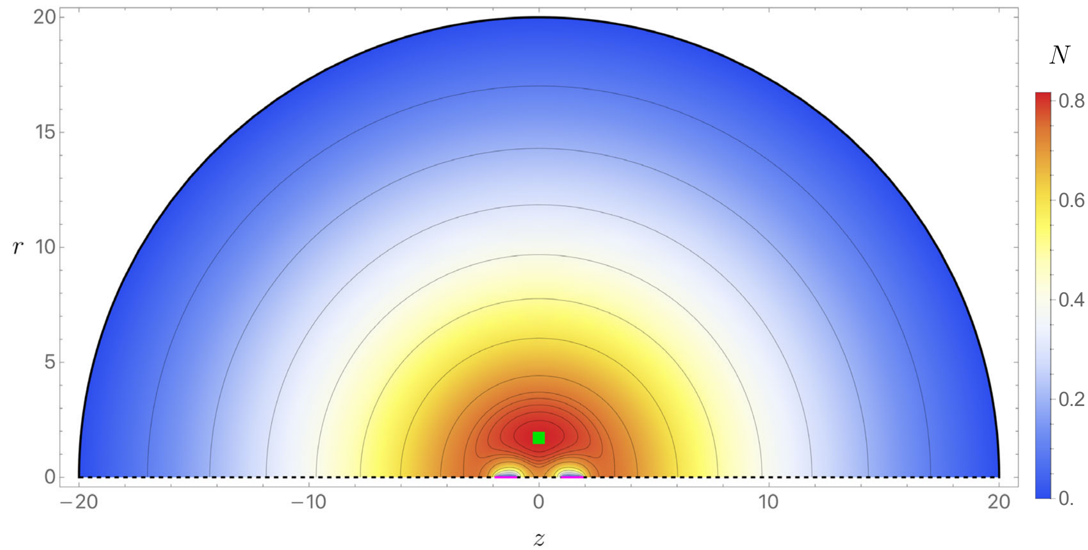

FIG. 3. Contour plot showing the level sets of the lapse function N. The cosmological horizon is the outer solid black semicircle. The horizon axes has the two black hole horizons as solid magenta lines, and the outer and inner axes in dashed black lines. The green square is where N takes its maximum value.

Following [96], we now consider the entropy, which must increase during time evolution. The blue dots in Fig. 2 show the entropy of the static binary as a function of the entropy S_c of the cosmological horizon. The black curve shows the entropy for the single Schwarzschild-de Sitter black hole (also known as the Kottler black hole). We see that for any given S_c, the single Schwarzschild black hole has higher entropy than the binary. This, together with the second law of thermodynamics, indicates that classically the binary can evolve toward the single black hole but not the other way around. The static black binary is therefore thermodynamically unstable. Beyond the classical level, when the black holes are small, Hawking radiation should however also play a role in this discussion.

The fact that (at least) two solutions exist for a given cosmological horizon entropy implies that the Schwarzschild-de Sitter black hole is not unique. This is the first counterexample to the no-hair conjecture [97] for pure gravity with a positive cosmological constant.

We now comment on the uniqueness theorems for de Sitter black holes [46–48] that would, under certain assumptions, rule out the existence of static de Sitter binaries. In [46], the level sets of the lapse function $N \equiv \sqrt{-g_{tt}}$ are assumed to be surface forming. In particular, this means that the level sets must consist only of 2D surfaces. In [47], the set $\mathrm{MAX}(N) = \{x \in \mathcal{M} : N(x) = N_{\max}\}$, where $N_{\max}$ is the maximum value of N in the manifold $\mathcal{M}$, is assumed to disconnect $\mathcal{M}$ into an inner region $\mathcal{M}_-$ and an outer region $\mathcal{M}_+$ with the same virtual mass. Our static binaries do not satisfy either of these assumptions. Indeed, in Fig. 3 we show the level sets N in our domain of integration for a typical solution (all of our solutions show the same qualitative behavior). The coordinates (r, z) are defined in the Supplemental Material. The cosmological horizon is represented by the outer solid black semicircle, the two black hole horizons are marked by solid magenta lines along the line $r = 0$, and the inner and outer portions of the symmetry axes are given by the dashed horizontal line. Finally, the green square marks the location of the maximum of N in $\mathcal{M}$. This maximum represents an S^1 on the manifold, which is not a 2D surface, and it also does not partition the manifold into two regions. Therefore, our static binaries fail to satisfy the assumptions in [46,47].

Finally, we comment on [48]. We believe that this work is not correct for a rather technical reason. Beginning with the Schwarzschild-de Sitter black hole, the authors in [48] argue that they can construct an asymptotically flat metric that is conformal to the original one, is topologically $S^1 \times S^2$ deprived of one point, and has zero Arnowitt-Deser-Misner (ADM) mass. If that were true, the rigidity statement in the positive mass theorem [98–102] would not only imply that the original metric is conformally flat, but also that $S^1 \times S^2$ with one point removed is diffeomorphic to $\mathbb{R}^3$, which is impossible.

Conclusions.—We constructed the first example of a multiple black hole solution within general relativity with a positive cosmological constant and established that the leading behavior of these solutions agrees (for small black holes) with estimates from Newton-Hooke theory. Based on thermodynamic considerations, we argued that these solutions are thermodynamically unstable. Because the configuration requires a delicate balance between gravitational attraction and cosmic expansion, we expect these solutions to also be dynamically unstable.

We have focused on the static configuration of two identical black holes, but our results and methods can be generalized. First, consider the case where the black holes have different masses. When one of the black holes is much smaller than the other, one can use the geodesic approximation to predict the existence of such a configuration. Indeed, one can easily confirm the existence of static orbits for timelike particles on a Schwarzschild-de Sitter black hole background, thus providing further evidence for the existence of this more general central configuration. Note that if [48] were correct, this asymmetric binary would also not exist.

We can also include rotation, which will introduce spin-spin interaction of the black holes. This opens the possibility of continuous nonuniqueness. Consider, for example, the case with two identical black holes rotating in opposite directions along the axis of symmetry. This configuration will have vanishing total angular momentum, and will thus be in the same class as the Schwarzschild-de Sitter black hole. Work in this direction is underway.

Perhaps more interestingly, because spin-spin interactions act on shorter length scales, they could provide a mechanism for stabilizing the binary. This possibility resembles the mechanism that provides stability for molecules. Work in this direction is underway.

We could also consider central configurations containing $N > 2$ static black holes in the static patch of de Sitter. These configurations can show interesting properties within the Newton-Hooke approximation. For instance, when $N \geq 13$, minimal energy central configurations do not lie on a regular polyhedron [49]. We thus expect the equivalent property within general relativity. The study of these configurations is within the reach of the numerical methods employed in this Letter.

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