

Error-Resilient Reversal of Quantum Chaotic Dynamics Enabled by Scramblons

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The emergence of the arrow of time in quantum many-body systems stems from the inherent tendency of Hamiltonian evolution to scramble quantum information and increase entanglement. While, in principle, one might counteract this temporal directionality by engineering a perfectly inverted Hamiltonian to reverse entanglement growth, such a scenario is fundamentally unstable because even minor imperfections in the backward evolution can be exponentially amplified, a hallmark of quantum many-body chaos. Therefore, successfully reversing quantum many-body dynamics demands a deep understanding of the underlying structure of quantum information scrambling and chaotic dynamics. In this Letter, by using solid-state nuclear magnetic resonance on a macroscopic ensemble of randomly interacting spins, we measure the out-of-time-ordered correlator and validate key predictions of scramblon theory, a universal theoretical framework for information scrambling. Crucially, this theory enables us to isolate and mitigate errors in the out-of-time-ordered correlator caused by imperfections in the backward evolution. As a result, this protocol uncovers the anticipated exponential behavior of quantum many-body chaos and extracts the quantum Lyapunov exponent in a many-body experimental system for the first time. Our results push the fundamental limits of dynamical reversibility of complex quantum systems, with implications for quantum simulation and metrology.

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Quantum information scrambling lies at the heart of thermalization in isolated quantum systems [1–5]. When a low-entanglement initial state evolves under a Hamiltonian $\hat{H}$, local information spreads across the entire system, becoming irretrievable via local measurements. At the same time, the entanglement entropy grows, rendering the system locally a thermal state. Intriguingly, one can envision a gedanken experiment: If we apply the reversed Hamiltonian $-\hat{H}$ to this thermal state, is it possible to recover the original low-entanglement state and its local information? This question echoes a profound analogy in black hole physics: whether information thrown in and thermalized by a black hole can ever be retrieved. Now, advances in quantum control techniques on various

synthetic quantum platforms attempt to turn such a thought experiment into a tangible ambition [6–31], raising an even deeper question: Can engineered inverted dynamics defy the arrow of time?

In realistic quantum systems, however, such an oversimplified scenario does not materialize. As illustrated in Fig. 1(a), even infinitesimal imperfections in the engineered reversed Hamiltonian can lead to an exponentially amplified error as evolution time increases, a hallmark of quantum many-body chaos [32–36]. Given that no practical implementation can perfectly realize $-\hat{H}$, the inherent chaotic nature of quantum many-body dynamics fundamentally prohibits the retrieval of information from a thermal state. The same reasoning extends analogously to the challenge of recovering information from a black hole. Recent experimental observations, spanning nuclear magnetic resonance (NMR) systems [6–14], trapped ions [15–18], superconducting qubits [19–23], ultracold atoms [24–30], and nitrogen vacancy centers [31], have

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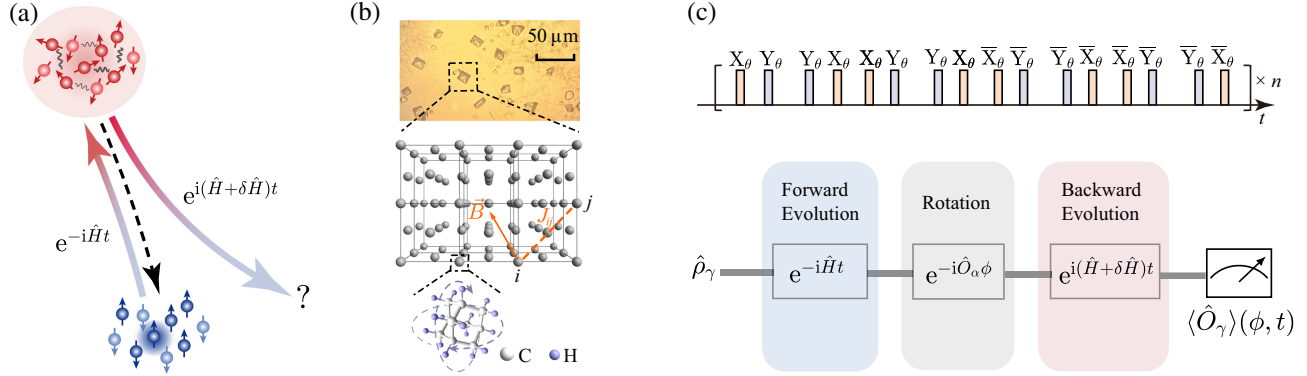


FIG. 1. Schematic of experimental protocol. (a) Chaotic quantum many-body dynamics: unitary evolution under $\hat{H}$ scrambles quantum information, transforming a low-entanglement initial state into a highly entangled state. Under time-reversed dynamics with perturbed Hamiltonian $-\hat{H} + \delta\hat{H}$, the system exhibits exponential deviation from perfect state recovery. (b) Microscopic structure of adamantane ($\text{C}_{10}\text{H}_{16}$) powder: each granule contains adamantane molecules arranged in a face-centered cubic lattice. Individual molecules consist of spin-1/2 ^1H nuclei and spinless ^{12}C atoms. (c) Top: Floquet pulse sequence converting the dipolar Hamiltonian equation (1) into the engineered form Eq. (2) [8,37]. Bottom: experimental sequence for MQC and OTOC measurements.

also directly observed the inevitable effects of imperfect backward evolution.

Although errors in time-reversed dynamics are inherently unavoidable, a potential solution is to postprocess experimental data to extrapolate an error-free limit. However, achieving an unbiased extrapolation remains highly challenging unless the imperfections meet the following two criteria: (i) the error model in the backward evolution must follow a universal behavior, independent of microscopic details; and (ii) this universal behavior can be captured by a theoretical framework that yields an analytical ansatz. Such an ansatz would enable both the reliable quantification of imperfections in the backward evolution and robust extrapolation to the error-free limit.

Remarkably, these two conditions can be simultaneously satisfied across a broad class of systems. Quantum information scrambling and many-body chaos are intimately linked, both characterized by the out-of-time-ordered correlator (OTOC) [32–36,38]. The recently developed scramblon theory has identified the scramblon, a novel type of collective excitation in quantum many-body systems with all-to-all connectivity, as the fundamental carrier of scrambling dynamics [39–42]. Analogous to phonons mediating density fluctuations or spin waves governing spin dynamics, scramblons dictate the spread of quantum information. The scramblon propagator grows exponentially with time, outpacing all other modes and dominating the late-time behavior of the OTOC, thereby resulting in fast information scrambling and a universal exponential behavior of the OTOC. Notably, in systems with holographic duality, scramblons in the boundary quantum theory correspond to gravitons in the bulk gravity [39,43].

In this Letter, we measure the OTOC in a solid-state NMR sample with a macroscopic number of randomly interacting spins, a system beyond the capabilities of

classical simulation. This can be achieved using a well-established protocol for measuring the multiple quantum coherence (MQC) spectrum [Fig. 1(c), bottom panel] [44,45]. The measurement involves the reverse evolution of this many-body system, which, of course, is imperfect. We utilize scramblon theory to derive a fitting ansatz for the measured OTOC, achieving good agreement with the experiment. Moreover, the scramblon theory imposes several strong constraints on the fitting parameters, and we verify that these constraints are well satisfied by the fitting results. In other words, the experimental data strongly validate the scramblon theory. With this fitting formula, we can extrapolate the measured OTOC to the error-free limit. We demonstrate that the OTOC after this error mitigation exhibits the anticipated long-time behavior, in sharp contrast to other methods explored in the previous literature [9,15,19].

Physical system and experimental protocol—The experiment is performed on a powdered adamantane ($\text{C}_{10}\text{H}_{16}$) sample, as illustrated in Fig. 1(b) and detailed in Supplemental Material [46]. Each adamantane molecule contains 16 nuclear spin-1/2 hydrogen nuclei, denoted by $\mathbf{S}_{im}$, where the index i labels the molecules and $m = 1, \dots, 16$ identifies individual spins within a molecule. Because of rapid molecular thermal motion, intramolecular spin interactions are averaged out, leaving only the intermolecular dipolar coupling as the dominant spin-spin interaction. The sample is placed in a uniform 9.4 T magnetic field, which aligns the spin quantization axes and induces fast Larmor precession of all spins. In the rotating frame and under the secular approximation, the effective dipolar Hamiltonian takes the form [66]

$$\hat{H}_d = \sum_{i < j, m, n} J_{ij} (-\hat{S}_{im}^x \hat{S}_{jn}^x - \hat{S}_{im}^y \hat{S}_{jn}^y + 2\hat{S}_{im}^z \hat{S}_{jn}^z). \quad (1)$$

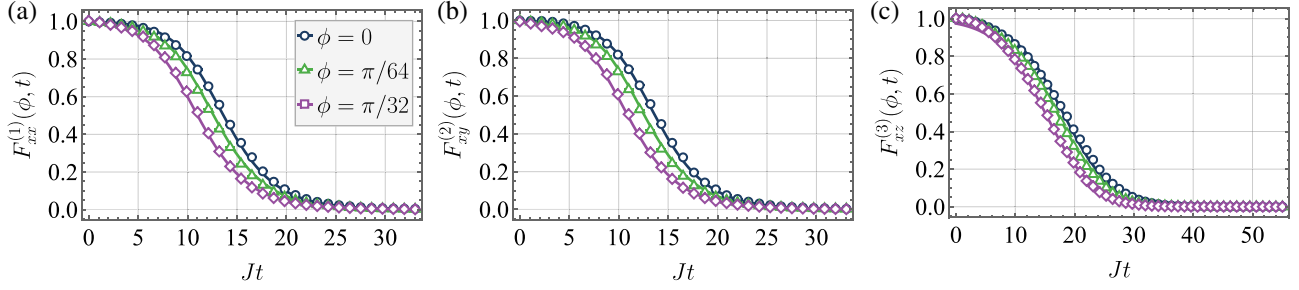


FIG. 2. Fitting experimental data with the scramblon ansatz. Three typical experimental data $F_{\alpha\gamma}^{(\lambda)}(\phi, t)$ for three different $\phi = 0, \pi/64$, and $\pi/32$. The superscript λ labels three different Hamiltonians, as described in the main text. The solid curves are fitting results obtained using the scramblon ansatz. Experimentally, the 95% confidence intervals determined from readout noise are approximately 10^{-4} , and, therefore, the error bars are contained within the data markers.

Here, the coupling strengths J_{ij} are treated as random variables with zero mean ($\overline{J_{ij}} = 0$) and variance $\overline{J_{ij}^2} = 4J^2/N$, where N is the total number of spins and $J \sim 2\pi \times 1460$ Hz [14]. This randomness of J_{ij} arises from the anisotropic nature of dipolar interaction and the random crystalline orientations in the powder sample [46].

Then, by applying periodic global radio-frequency pulse sequences to rotating spins, as depicted in Fig. 1(c), a more general effective Hamiltonian can be engineered via the Floquet-Magnus expansion as $\hat{H}_{\text{eff}} = \hat{H}_0 + \hat{H}_1$. $\hat{H}_0$ represents the leading-order contribution in the expansion, which can be generally written as

$$\hat{H}_0 = \sum_{i < j, m, n} \sum_{\mu, \nu} J_{ij} \xi_{\mu\nu} \hat{S}_{im}^{\mu} \hat{S}_{jn}^{\nu}, \quad (2)$$

where $\mu, \nu = x, y, z$. The protocol generating Eq. (2) imposes specific constraints on the coefficients $\xi_{\mu\nu}$, namely, the traceless condition $\sum_{\mu=x,y,z} \xi_{\mu\mu} = 0$ and the symmetry condition $\xi_{\mu\nu} = \xi_{\nu\mu}$. An important observation is that if a set $\{\xi_{\mu\nu}\}$ satisfies these constraints, so does the set $\{-\xi_{\mu\nu}\}$. This implies that Floquet engineering always enables reversal of $\hat{H}_0$. However, the higher-order terms, denoted by $\hat{H}_1$, generally cannot be simultaneously reversed. Therefore, without loss of generality, we denote the forward evolution as governed by $\hat{H} = \hat{H}_0 + \hat{H}_1$ and the backward evolution as governed by $-\hat{H}_0 - \hat{H}_1' = -\hat{H} - \delta\hat{H}$. Here, $\delta\hat{H}$, representing the difference between $\hat{H}_1$ and $\hat{H}_1'$, is the imperfection in the reversed dynamics. In this system, $\hat{H}_1$ and $\delta\hat{H}$ are typically on the order of a few percent relative to $\hat{H}_0$ [14,46].

The measurement protocol is schematically illustrated in Fig. 1(c). We begin by defining the total spin operator along the γ axis as $\hat{O}_{\gamma} = \sum_{im} \hat{S}_{im}^{\gamma}$, where $\gamma \in \{x, y, z\}$. The system is initialized in a weakly polarized state described by the density matrix $\hat{\rho}_{\gamma} \propto \mathbb{1} + \epsilon \hat{O}_{\gamma}$, with $\epsilon \ll 1$ characterizing the small polarization. Following forward time evolution under the Hamiltonian $\hat{H}$, the density matrix

transforms as $\hat{\rho}_{\gamma}(t) = e^{-i\hat{H}t} \hat{\rho}_{\gamma} e^{i\hat{H}t}$. Subsequently, we implement a uniform spin rotation by an angle ϕ around the α axis ($\alpha \in \{x, y, z\}$), yielding the transformed density matrix: $\hat{\rho}_{\alpha\gamma}(\phi, t) = e^{-i\hat{O}_{\alpha}\phi} e^{-i\hat{H}t} \hat{\rho}_{\gamma} e^{i\hat{H}t} e^{i\hat{O}_{\alpha}\phi}$. This is followed by backward evolution as described above. Finally, we measure the total magnetization along the γ direction $\langle \hat{O}_{\gamma} \rangle$, which equals the correlation function $\epsilon F_{\alpha\gamma}(\phi, t)$ defined as

$$F_{\alpha\gamma}(\phi, t) = \text{Tr}[\hat{O}_{\gamma}(0) \hat{\mathcal{V}}^{\dagger}(t) e^{-i\phi \hat{O}_{\alpha}(t)} \hat{O}_{\gamma}(0) e^{i\phi \hat{O}_{\alpha}(t)} \hat{\mathcal{V}}(t)]. \quad (3)$$

Here, $\hat{\mathcal{V}}(t) \equiv \exp(-i\mathbf{T} \int_0^t dt' \delta\hat{H}(t'))$ represents the time-evolution operator in the interaction picture, with $\mathbf{T}$ denoting time ordering. All operators $\hat{O}(t)$ and $\delta\hat{H}(t)$ evolve according to the Heisenberg picture under $\hat{H}$. For normalization, we divide $F_{\alpha\gamma}(\phi, t)$ by $\mathcal{C} = \langle \hat{O}_{\gamma}^2 \rangle = N/4$ to ensure $F_{\alpha\gamma}(0, 0) = 1$. The MQC spectrum can be obtained through the Fourier transformation of $F_{\alpha\gamma}(\phi, t)$ with respect to ϕ [44,45].

Validating the scramblon theory—We now highlight the relationship between $F_{\alpha\gamma}(\phi, t)$ and OTOCs through two limits: (i) For perfect reversed dynamics ($\delta\hat{H} = 0$), $F_{\alpha\gamma}(\phi, t)$ reduces to

$$F_{\alpha\gamma}^{(0)}(\phi, t) = \text{Tr}[\hat{O}_{\gamma}(0) e^{-i\phi \hat{O}_{\alpha}(t)} \hat{O}_{\gamma}(0) e^{i\phi \hat{O}_{\alpha}(t)}], \quad (4)$$

representing an OTOC of operators $e^{i\phi \hat{O}_{\alpha}(t)}$ and $\hat{O}_{\gamma}$; (ii) at $\phi = 0$, it becomes

$$F_{\alpha\gamma}(0, t) = \text{Tr}[\hat{O}_{\gamma}(0) \hat{\mathcal{V}}^{\dagger}(t) \hat{O}_{\gamma}(0) \hat{\mathcal{V}}(t)], \quad (5)$$

and this OTOC, also known as the Loschmidt echo, probes imperfections in the reversed dynamics. Both cases can be computed by scramblon theory [42,67], strongly suggesting that the general behavior of $F_{\alpha\gamma}(\phi, t)$, although more challenging to derive, can also be captured by scramblon theory. Indeed, as we show in Supplemental Material [46], an ansatz for $F_{\alpha\gamma}(\phi, t)$ in the small- ϕ regime can be derived as

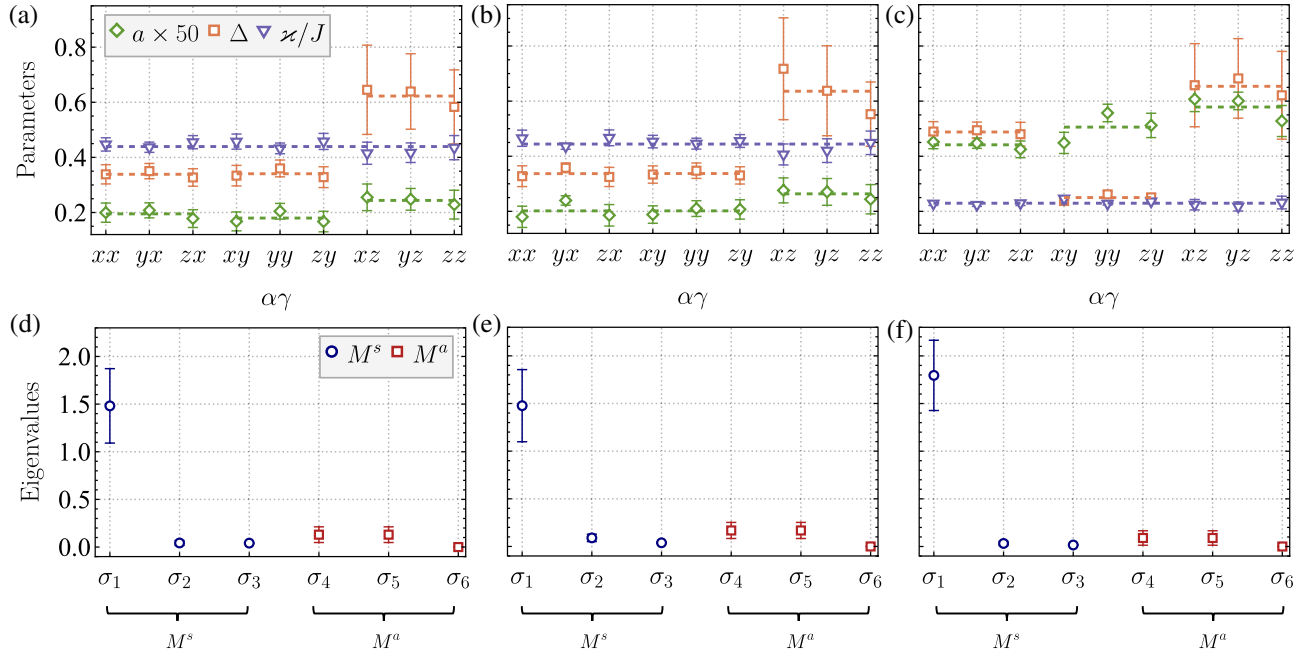


FIG. 3. Verifying the predictions of scramblon theory. The three columns correspond to the fitting parameters extracted from three different Hamiltonians: $\hat{H}^{(1)}$, $\hat{H}^{(2)}$, and $\hat{H}^{(3)}$. (a)–(c) in the upper row display the fitting parameters for different values of α and γ , with green diamonds representing $a \times 50$, yellow squares representing Δ , and blue triangles representing κ/J . The dashed lines represent κ averaged over α and γ , and a and Δ averaged over α . (d)–(f) in the lower row show the eigenvalues σ_1 , σ_2 , and σ_3 for the symmetric matrix M^s and the absolute value of eigenvalues σ_4 , σ_5 , and σ_6 for the antisymmetric matrix M^a . The error bars on the fitting parameters represent the 95% confidence interval, including the standard deviation from each fitting residual σ_{res} and the uncertainty arising from the choice of fitting range σ_{range} [46].

$$F_{\alpha\gamma}(\phi, t) = \frac{1}{(1 + ae^{\kappa t} + b\phi^2 e^{\kappa t})^{2\Delta}}, \quad (6)$$

where $\{a, b, \kappa, \Delta\}$ are all fitting parameters that depend on α and γ . The $\phi = 0$ limit of Eq. (6) reproduces the Loschmidt echo [Eq. (5)], with $a \neq 0$ quantifying the reversal imperfection. We note that a recent theoretical work also discussed a similar issue in a specific solvable model [68].

We investigate three representative chaotic Hamiltonians: (i) $\hat{H}_0^{(1)}$ with $\xi_{xx} = -\xi_{yy} = 0.4$, (ii) $\hat{H}_0^{(2)}$ with $\xi_{xy} = \xi_{yx} = 0.4$, and (iii) $\hat{H}_0^{(3)}$ with $(\xi_{xx}, \xi_{yy}, \xi_{zz}) = (0.2, -0.225, 0.025)$, while all other $\xi_{\mu\nu} = 0$. For each Hamiltonian $\hat{H}_0^{(\lambda)}$, we perform measurements for all nine possible combinations of $\alpha, \gamma \in \{x, y, z\}$ axes, yielding a total of $3 \times 9 = 27$ distinct cases. For each case, we measure three different ϕ angles and fit the three curves jointly with a single scramblon ansatz, Eq. (6). We find that all data are excellently captured by the scramblon ansatz Eq. (6), with three representative fits shown in Fig. 2. We quantify the goodness of fit using the adjusted R^2 , as detailed in Supplemental Material [46]. Discrepancies between experiment and theory may arise from the invalidity of the scramblon theory at very early times $e^{\kappa t} \lesssim 1$.

The scramblon theory not only provides the fitting ansatz, but also imposes strong constraints on the parameters. For a given Hamiltonian, while the fitting yields parameter sets indexed by α and γ , the theory predicts the following: (1) κ must be independent of both α and γ , since the system is dominated by a single scramblon mode and κ characterizes the intrinsic growth rate of this mode. (2) a and Δ cannot depend on α , as they are retained in the $\phi = 0$ limit where no α -axis spin rotation is applied. (3) The matrix $M_{\alpha\gamma} \equiv b_{\alpha\gamma} \Delta_\gamma$ must be symmetric and rank-1, reflecting the symmetric coupling of a single scramblon mode to both $\hat{O}_\alpha$ and $\hat{O}_\gamma$ operators.

Figure 3 demonstrates how we verify these three constraints for our three Hamiltonians. Figures 3(a)–3(c) display the values of a , Δ , and κ for all nine measurement combinations per Hamiltonian, with dashed lines indicating (i) κ averaged over γ and α and (ii) α -averaged a and Δ values that depend only on γ . Most cases show only a few percent deviations from averaged values, with a maximum deviation of $\sim 16\%$ occasionally, confirming the first two constraints. To test the third constraint, we decompose M into symmetric [$M^s = (M + M^T)/2$] and antisymmetric [$M^a = (M - M^T)/2$] components. Eigenvalues shown in Figs. 3(d)–3(f) reveal that M^s exhibits only one dominant eigenvalue, and all eigenvalues of M^a are negligible, validating the rank-1 symmetric requirement for M . It is

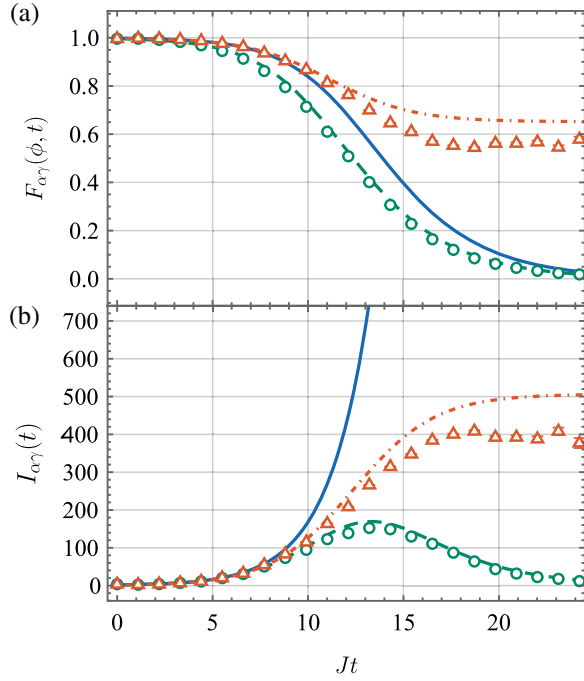


FIG. 4. Error mitigation of the reversed quantum many-body dynamics. (a) $F_{\alpha\gamma}(\phi, t)$ and (b) $I_{\alpha\gamma}(t)$ defined in Eq. (8). As an example, we show $\alpha, \gamma = z, y$, $\phi = \pi/64$, and with Hamiltonian $\hat{H}_0^{(1)}$. The blue solid line results from the scramblon ansatz by setting $a = 0$. The red triangles are error mitigated by $\tilde{F}_{\alpha\gamma}(\phi, t)$, and green circles are raw data without error mitigation. The red dash-dotted line and the green dashed line are obtained from the scramblon ansatz of $\tilde{F}_{\alpha\gamma}(\phi, t)$ and $F_{\alpha\gamma}(\phi, t)$ (without setting $a = 0$). The experimental data points in (b) were extracted by fitting the corresponding data for $F_{\alpha\gamma}(\phi, t)$ with a sixth-order polynomial. The error bars represent the standard deviations.

notable that a strong correspondence between the results for $H_0^{(1)}$ and $H_0^{(2)}$ is observed. This is anticipated, as $H_0^{(1)}$ can be transformed into $H_0^{(2)}$ via a global rotation $e^{-i\hat{\sigma}_z\pi/4}$.

Recovering exponential behavior of OTOC and extracting the Lyapunov exponent—The excellent agreement with experimental data strongly validates the scramblon ansatz. Within this framework, the parameter a fully captures imperfections in the time-reversed dynamics, as evidenced by $F_{\alpha\gamma}(0, t) \equiv 1$ when $a = 0$. This allows an error mitigation by setting $a = 0$ in Eq. (6) while keeping other fitted parameters unchanged [blue curve in Fig. 4(a)]. For comparison, we examine an alternative mitigation approach from previous literature that normalizes $F_{\alpha\gamma}(\phi, t)$ by $F(0, t)$ to define $\tilde{F}_{\alpha\gamma}(\phi, t) = F_{\alpha\gamma}(\phi, t)/F_{\alpha\gamma}(0, t)$ [red triangles in Fig. 4(a)] [9,15,19,69]. Within the scramblon formalism, this yields

$$\tilde{F}_{\alpha\gamma}(\phi, t) = \left(\frac{1 + ae^{\chi t}}{1 + ae^{\chi t} + b\phi^2 e^{\chi t}} \right)^{2\Delta}. \quad (7)$$

Different approaches are compared in Fig. 4(a).

To assess the reliability of different approaches, we compute $I_{\alpha\gamma}(\phi)$ from $F_{\alpha\gamma}(\phi, t)$ as

$$I_{\alpha\gamma}(t) \equiv -\partial_\phi^2 F_{\alpha\gamma}(\phi, t)|_{\phi=0}, \quad (8)$$

which for perfect time reversal ($\delta\hat{H} = 0$) corresponds to the OTO commutator $\langle |[\hat{O}_\alpha(t), \hat{O}_\gamma]|^2 \rangle$ and exhibits exponential growth, reflecting the rapid increase of the operator size $\hat{O}_\alpha(t)$ under Heisenberg evolution before reaching the scrambling time. Analysis based on the scramblon ansatz reveals that (i) the raw data [Eq. (6)] gives $I_{\alpha\gamma}(t) = 4b\Delta e^{\chi t}/(1 + ae^{\chi t})^{1+2\Delta}$, which decays at late times due to $\Delta > 0$ [green dashed line in Fig. 4(b)]; (ii) the normalized approach [Eq. (7)] yields $I_{\alpha\gamma}(t) = 4b\Delta e^{\chi t}/(1 + ae^{\chi t})$, saturating to $4b\Delta/a$ at long times [red dash-dotted line in Fig. 4(b)]; and (iii) only our $a = 0$ mitigation produces the physically expected exponential growth $4b\Delta e^{\chi t}$ [blue solid line in Fig. 4(b)], where χ emerges as the quantum Lyapunov exponent. It is also remarkable that only our $a = 0$ mitigated result $I_{\alpha\gamma}(t)$ exhibits the symmetric and rank-1 property predicted by the scramblon theory for systems with time-reversal symmetry, because there is a single dominant scramblon mode and its coupling to both $\hat{O}_\alpha$ and $\hat{O}_\gamma$ operators is symmetric (see Supplemental Material [46] for more experimental data on all combinations of α and γ). These distinct behaviors are quantitatively compared with experimental data. To the best of our knowledge, χ shown in Fig. 3 serves as the first exact experimental determination of the Lyapunov exponent in a quantum many-body system with macroscopic degrees of freedom.

Conclusion—In conclusion, we develop an ansatz based on scramblon theory to describe reversed quantum many-body dynamics in the presence of imperfections. We validate this ansatz by fitting it to experimental data from an NMR system with macroscopically randomly interacting spins, where the stringent constraints imposed by scramblon theory are convincingly verified by the fitting outputs. These results suggest that the applicability of the scramblon theory extends beyond idealized all-to-all interacting models, remaining valid in realistic systems with microscopic long-range interactions. The application of the scramblon theory allows us to mitigate the effects of imperfections and recover error-resilient reversed dynamics. Notably, time-reversal-based exponential sensitivity has also been recently exploited to enhance quantum metrology in systems such as cold atoms in cavities [24,25] and nitrogen vacancy centers [31], where imperfections also suppress this sensitivity at long evolution times. Our theory may be extended to such quantum metrology applications, potentially enabling further improvements in sensitivity.

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Data availability—The data that support the findings of this article are openly available [70].

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End Matter

Hamiltonian engineering—An effective Hamiltonian $\hat{H}_{\text{eff}}$ can be engineered from the dipolar Hamiltonian $\hat{H}_d$ [Eq. (1)], through applying the radio-frequency (rf) pulse sequences periodically. The rf pulse train was constructed from a basic four-pulse sequence applied from left to right in chronological order [8,37]:

$$(\tau_1, \mathbf{X}_\theta, \tau_2, \mathbf{Y}_\theta, \tau_3, \mathbf{Y}_\theta, \tau_4, \mathbf{X}_\theta, \tau_5), \quad (\text{A1})$$

where $\mathbf{X}_\theta = e^{-i\hat{\sigma}_z\theta}e^{-i\hat{\sigma}_x\pi/2}e^{i\hat{\sigma}_z\theta}$ denotes a $\pi/2$ pulse with azimuthal phase θ ; the operator $\mathbf{Y}_\theta$ is defined similarly for a rotation about the y axis. The terms $\{\tau_g\}$ represent the interpulse delays.

The average Hamiltonian can be calculated in a toggling frame following the discrete rotations induced by the pulses [66]. In this frame, the Hamiltonian becomes a piecewise-constant function in time intervals. During a given interval τ_g , the toggling-frame Hamiltonian is

$$\hat{H}_{\text{togg}}^{\{g\}} = \sum_{i < j, m, n} J_{ij} [3(\hat{S}_{im} \cdot \mathbf{u}_g)(\hat{S}_{jn} \cdot \mathbf{u}_g) - \hat{S}_{im} \cdot \hat{S}_{jn}], \quad (\text{A2})$$

where the unit vector $\mathbf{n}_g$ is determined by the preceding pulses. We designed the pulse intervals τ_g and phases θ such that the zeroth-order average Hamiltonian matched the target Hamiltonian $\hat{H}_0$ [Eq. (2)], which is expressed as $\hat{H}_0 = (1/T) \sum_{g=1}^5 \tau_g \hat{H}_{\text{togg}}^{\{g\}}$, where $T = \sum_{g=1}^5 \tau_g$ is the total duration of the pulse sequence. The resulting anisotropic tensor ξ has elements given by $\xi_{\mu\nu} = \sum_{g=1}^5 \xi_{\mu\nu}^{\{g\}} \tau_g / T$, where $\xi_{\mu\nu}^{\{g\}} = 3u_g^\mu u_g^\nu - \delta_{\mu\nu}$ is the term during the interval τ_g .

The elements of the time-averaged tensor ξ are subject to the following constraints. The diagonal terms are traceless and bounded:

$$\sum_{\mu} \xi_{\mu\mu} = 0, \quad -1 \leq \xi_{\mu\mu} \leq 2 (\text{constraint 1}). \quad (\text{A3})$$

The off-diagonal terms are symmetric and are also bounded:

$$\xi_{\mu\nu} = \xi_{\nu\mu}, \quad |\xi_{\mu\nu}| \leq 3/2 (\text{constraint 2, for } \mu \neq \nu). \quad (\text{A4})$$

Experimental limitations, specifically a finite pulse width (2 μs in our case), prevent the delays τ_g from reaching zero, which limits the accessible range of the tensor elements. For a cycle period of $T = 30 \mu\text{s}$, the achievable ranges were restricted to $-0.6 \leq \xi_{\mu\mu} \leq 1.2$ and $|\xi_{\mu\nu}| \leq 0.9$ ($\mu \neq \nu$). By tuning $\{\tau_g\}$ and θ , we successfully engineered all target $\hat{H}_0$ with both positive and negative signs. Further details are provided in Supplemental Material [46].

The forward evolution is captured by the total effective Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_1$, while the one for the backward evolution is $-\hat{H}_0 - \hat{H}_1$. The deviation from ideal time reversal is $\delta\hat{H} = \hat{H}_1' - \hat{H}_1$, arising from the higher-order Floquet-Magnus expansions and experimental control imperfections such as finite pulse width, flip-angle errors, and field fluctuations. We quantify the magnitude of $\delta\hat{H}$ with an error metric $\varepsilon \equiv \sqrt{\text{Tr}[(\delta\hat{H})^2]/\text{Tr}[(\hat{H}_0)^2]}$. Numerical simulations up to the second order of Floquet-Magnus expansion and with $N = 8$ spins suggest that $\varepsilon \sim 3\%$ for the three different $\hat{H}_0$ investigated (see Supplemental Material [46] for details).

Scramblon field theory—Scramblon field theory provides an analytical framework for deriving various generalizations of OTOC [39–42]. For example, for a general OTOC involving simple operators $\hat{W}$ and $\hat{V}$, it reads

$$\begin{aligned} F_{\hat{W}, \hat{V}} &= \langle \hat{W}^\dagger(t_1) \hat{V}^\dagger(t_3) \hat{W}(t_2) \hat{V}(t_4) \rangle \\ &= \frac{1}{2} \text{diagram 1} + \frac{1}{2} \text{diagram 2} + \frac{1}{2} \text{diagram 3} + \dots \\ &= \sum_{k=0}^{\infty} \frac{(-\lambda)^k}{k!} \Upsilon_{\hat{W}}^{\text{R},k}(t_1 - t_2) \Upsilon_{\hat{V}}^{\text{A},k}(t_3 - t_4). \end{aligned} \quad (\text{B1})$$

The vertices $\Upsilon_{\hat{W}}^{\text{R},k}$ and $\Upsilon_{\hat{V}}^{\text{A},k}$, represented by the gray dots, are the scattering vertices between operators $\hat{W}$, $\hat{V}$, and k scramblons in the future and the past, respectively. To ensure that the scramblon enters the free collective mode region, we set $t_1 \approx t_2 \gg t_3 \approx t_4$. The term $\lambda = C^{-1} e^{\lambda[(t_1+t_2-t_3-t_4)/2]}$ is the propagator for the scrambling modes, depicted by the wavy lines. The prefactor C is proportional to the number of degrees of freedom N .

As discussed in the main text, two types of OTOCs from Eqs. (4) and (5) are involved in the calculation of the correlator $F_{\alpha\gamma}(\phi, t)$. This implies that both operators $\hat{O}_\alpha$ and $\delta\hat{H}$ can exchange scramblons with the operator $\hat{O}_\gamma$. Leaving further details to Supplemental Material [46], we show that these contributions can be captured by the equation

$$F_{\alpha\gamma}(\phi, t) = f_{\hat{O}_\gamma}^{\text{A}}(\tilde{\Upsilon}_{\delta\hat{H}}^{\text{R},1} C^{-1} \mathbf{x}^{-1} e^{\mathbf{x}t} + \phi^2 \Upsilon_{\hat{O}_\alpha}^{\text{R},1}(0) C^{-1} e^{\mathbf{x}t}, 0) C^{-1}. \quad (\text{B2})$$

This is the central prediction of the scramblon theory. The constant $\tilde{\Upsilon}_{\delta\hat{H}}^{\text{R},1} = \int_{-\infty}^{\infty} dt' \Upsilon_{\delta\hat{H}}^{\text{R},1}(t')$ is the reduced vertex

function and $\mathcal{C}$ is the normalization constant. Here, we have introduced the auxiliary function $f^{\text{R/A}}(x, t) \equiv \sum_{k=0}^{\infty} [(-x)^k/k!] \Upsilon^{\text{R/A},k}(t)$ to simplify the result. In the argument of Eq. (B2), the first term corresponds to the contribution from imperfection, while the second term arises from the uniform spin rotation.

In general, obtaining an analytical form for $f_{\hat{\mathcal{O}}_\gamma}^{\text{R}}(x, t)$ in randomly interacting spin systems is difficult. However, all known solvable models, such as the large- q Sachdev-Ye-Kitaev (SYK), Brownian SYK, and Brownian circuit models, share a similar ansatz for $f_{\hat{\mathcal{O}}_\gamma}^{\text{R}}(x, t)$ as [42]:

$$f_{\hat{\mathcal{O}}_\gamma}^{\text{R}}(x, 0) = f_{\hat{\mathcal{O}}_\gamma}^{\text{A}}(x, 0) = \mathcal{C}(1 + c_\gamma x)^{-2\Delta_\gamma}, \quad (\text{B3})$$

where the parameters c_γ and the scaling dimension Δ_γ depend on both the choice of model and the detection direction γ . Under this assumption, Eq. (B2) reduces to the form of Eq. (6), with the following identification:

$$a_\gamma = c_\gamma C^{-1} \chi^{-1} \tilde{\Upsilon}_{\delta \hat{H}}^{\text{R},1}, \quad b_{\alpha\gamma} = \Upsilon_{\hat{\mathcal{O}}_\alpha}^{\text{R},1}(0) c_\gamma C^{-1}. \quad (\text{B4})$$

Furthermore, the vertex function of the spin operators is related to the ansatz in Eq. (B3) by the relation $\Upsilon_{\hat{\mathcal{O}}_\gamma}^{\text{R/A},1}(t) = -\partial_x f_{\hat{\mathcal{O}}_\gamma}^{\text{R/A}}(x, t)|_{x=0}$, yielding $\Upsilon_{\hat{\mathcal{O}}_\gamma}^{\text{R/A},1} = 2\Delta_\gamma c_\gamma$. We can then immediately verify all the constraints introduced in the main text. In particular, we find that $M_{\alpha\gamma} = b_{\alpha\gamma} \Delta_\gamma = 2\Delta_\alpha c_\alpha \Delta_\gamma c_\gamma \mathcal{C} C^{-1}$ is a symmetric matrix in α and γ , which must satisfy $\text{rank}(M_{\alpha\gamma}) = 1$.